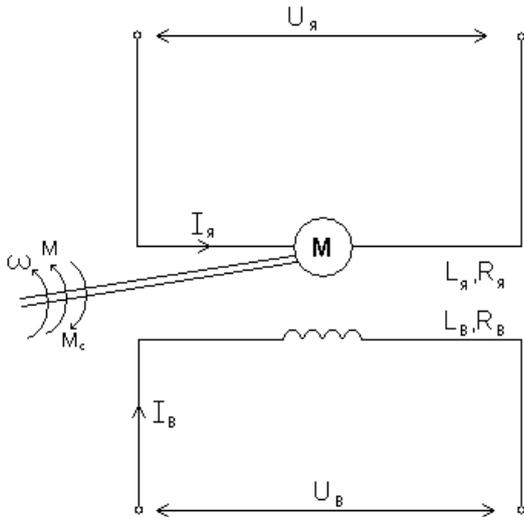


Laboratory work 3

Simulation of a separately excited dc motor operation in different modes and study of its characteristics

3.1. Objective

Mathematical description and simulation of a separately excited dc motor operation under various conditions, analysis of its stability with different criteria, assessment of its time domain specifications and steady-state errors.



3.2 Initial data:

Rated capacity P_{rat} , kW

Rounds per minute n_r , rpm

Load current I_{load} , A

Resistance of the armature winding and its additional poles $R_{A\Sigma} = r_a + r_{ap}$, Ω

Number of the armature parallel circuits $2p$

Magnetic flux of the pole Φ , mWb

Inertia of the armature J , $\text{kg}\cdot\text{m}^2$.

Figure 3.1 – Excitation of a separately excited dc motor

3.3 Mathematical description and transfer function of a separately excited dc motor

The system of differential equations describing electrical and mechanical balance of the motor is the following:

$$\begin{cases} U_{EX} = I_{EX} R_{EX} + L_{EX} \frac{dI_{EX}}{dt} & (1') \\ U_A = I_A R_{A\Sigma} + L_A \frac{dI_A}{dt} + E_M & (2') \\ E_M = K\Phi\omega & (3') \\ M = K\Phi I_A & (4') \\ J \frac{d\omega}{dt} = M - M_R & (5') \end{cases} \quad (3.1)$$

where U_{EX} is the motor excitation winding voltage, V;

I_{EX} is the motor excitation winding current, A;
 R_{EX} is the motor excitation winding resistance, Ω ;
 L_{EX} is the motor excitation winding inductance, H;
 U_A is the motor voltage, V;
 I_A is the motor armature current, A;
 L_A is the motor armature winding inductance, H;
 $R_{A\Sigma}$ is Resistance of the armature winding and its additional poles, Ω ;
 E_M is the motor emf, V · s ;
 K is a design coefficient;
 ω is the motor rotational speed, rpm;
 Φ is magnetic flux, Wb;
 M is the motor torque, N · m;
 M_R is resistant torque, N · m;
 I_S is static load current, A
 J is inertia of the armature, $kg \cdot m^2$.

The following simplifications are made to build a mathematical model of a separately excited dc motor:

1. The system is absolutely rigid, that is the motor has one degree of freedom;
2. The rotational elements are of identical mass;
3. The static torque on the shaft is constant;
4. The armature reaction is compensated;
5. The motor shaft rotational speed versus the armature voltage U_A is linear;
6. Magnetic flux Φ is constant.

Substituting equation (3') to equation (2') in system (3.1), we obtain:

$$U_A = I_A \cdot R_{A\Sigma} + L_A \frac{dI_A}{dt} + k\Phi\omega. \quad (3.2)$$

Substituting (4') to (5'), we obtain the following expression

$$J \frac{d\omega}{dt} = M - M_R = K\Phi I_A - M_R, \quad (3.3)$$

which allows obtaining the motor armature current I_A

$$I_A = \frac{M_R}{K\Phi} + \frac{J}{K\Phi} \cdot \frac{d\omega}{dt}. \quad (3.4)$$

and its derivative

$$\frac{dI_A}{dt} = \frac{J}{K\Phi} \cdot \frac{d^2\omega}{dt^2} \quad (3.5)$$

to substitute them to (3.2):

$$U_A = \frac{R_{A\Sigma}}{K\Phi} \cdot M_R + \frac{J R_{A\Sigma}}{K\Phi} \cdot \frac{d\omega}{dt} + \frac{J L_A}{K\Phi} \cdot \frac{d^2\omega}{dt^2} + K\Phi\omega$$

$$\frac{J L_A}{(K\Phi)^2} \cdot \frac{d^2\omega}{dt^2} + \frac{J R_{A\Sigma}}{(K\Phi)^2} \cdot \frac{d\omega}{dt} + \omega = \frac{1}{K\Phi} U_A - \frac{R_{A\Sigma}}{(K\Phi)^2} \cdot M_R. \quad (3.6)$$

Let us introduce the following notation:

– electromechanical time constant

$$T_M = \frac{J \cdot R_{A\Sigma}}{(K\Phi)^2}; \quad (3.7)$$

– electromagnetic time constant of the armature circuit

$$T_A = \frac{L_A}{R_{A\Sigma}}; \quad (3.8)$$

– transfer factor over the motor armature voltage

$$k_M = \frac{1}{K\Phi}; \quad (3.9)$$

– transfer factor over the motor static torque (disturbance)

$$k_S = \frac{R_{A\Sigma}}{(K\Phi)^2}. \quad (3.10)$$

Substituting (3.7) – (3.10) to equation of the motor static equilibrium (3.6), we obtain a differential equation of the motor relating the controlled parameter ω to the task U_A and disturbance M_R actions:

$$T_A \cdot T_M \cdot \frac{d^2\omega}{dt^2} + T_M \cdot \frac{d\omega}{dt} + \omega = k_M \cdot U_A - k_S \cdot M_R \quad (3.11)$$

In Laplace transforms:

$$T_A \cdot T_M \cdot S^2 \cdot \omega(S) + T_M \cdot S \cdot \omega(S) + \omega(S) = k_M \cdot U_A(S) - k_S \cdot M_R(S) \quad (3.12)$$

or

$$(T_A \cdot T_M \cdot S^2 + T_M \cdot S + 1) \cdot \omega(S) = k_M \cdot U_A(S) - k_S \cdot M_R(S)$$

The transfer functions of the separately excited DC motor for the task and the disturbance signals, respectively, are:

$$W_{U_A}(S) = \frac{\omega(S)}{U_A(S)} = \frac{k_M}{T_A \cdot T_M \cdot S^2 + T_M \cdot S + 1} \quad (3.13)$$

$$W_{M_R}(S) = \frac{\omega(S)}{M_R(S)} = \frac{k_S}{T_A \cdot T_M \cdot S^2 + T_M \cdot S + 1} \quad (3.14)$$

3.4 Block diagram of a separately excited dc motor

An algorithmic block diagram represents differential equations written in Laplace transforms. System of differential equations (3.1) in Laplace transforms is:

$$\begin{cases} U_A(S) = R_{A\Sigma} I_A(S) + L_A S I_A(S) + E_M(S) \\ JS\omega(S) = K\Phi(I_A(S) - I_S(S)) \end{cases},$$

or

$$\begin{cases} U_A(S) - E_M(S) = (R_{A\Sigma} + L_A S) I_A(S) \\ JS\omega(S) = K\Phi(I_A(S) - I_S(S)) \end{cases} \quad (3.15)$$

From the first equation of system (3.15), we obtain the transfer function for the electric circuit of the motor:

$$W(S) = \frac{I_A(S)}{U_A(S) - E_M(S)} = \frac{1}{R_{A\Sigma} + L_A S} = \frac{1/R_{A\Sigma}}{\frac{L_A}{R_{A\Sigma}} S + 1} = \frac{1/R_{A\Sigma}}{T_A S + 1}. \quad (3.16)$$

From the other equation of system (3.15), we obtain the transfer function for the mechanical circuit of the motor:

$$W(S) = \frac{\omega(S)}{I_A(S) - I_S(S)} = \frac{K\Phi}{JS} = \frac{K\Phi \cdot J \frac{R_{A\Sigma}}{(K\Phi)^2}}{JS \cdot T_M} = \frac{\frac{R_{A\Sigma}}{K\Phi}}{T_M S} \quad (3.17)$$

The algorithmic block diagram of a separately excited dc motor is:

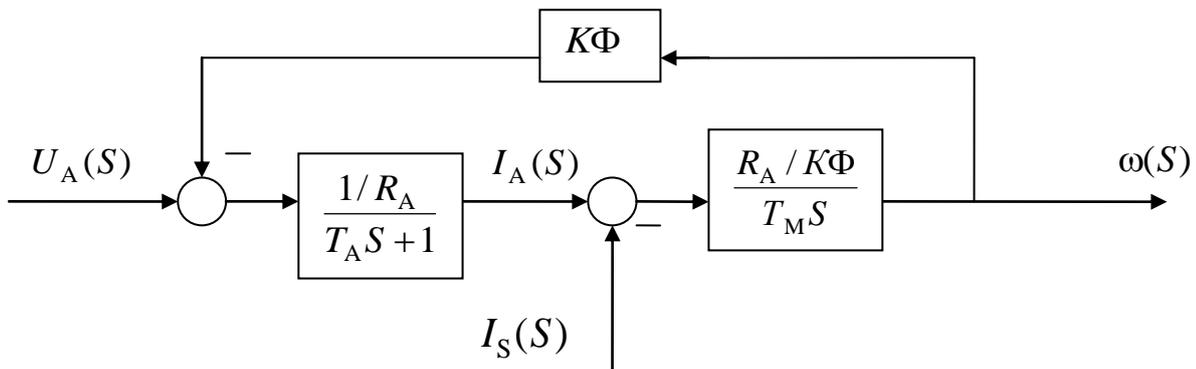


Figure 3.2 – The algorithmic block diagram of a separately excited dc motor

A Simulink model for simulating the motor output behavior under various operation conditions is the following:

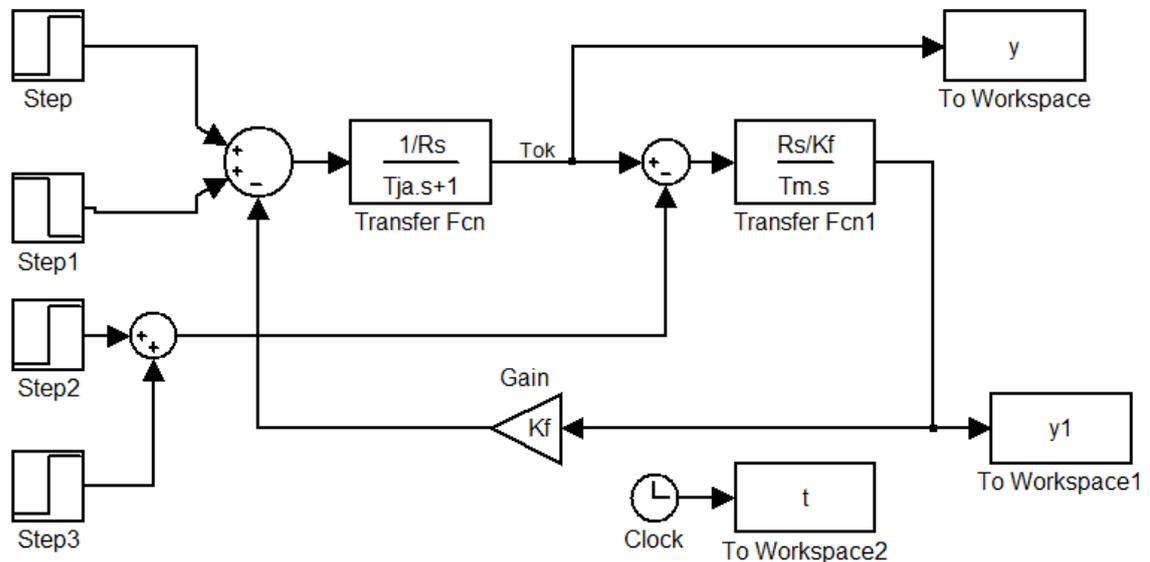


Figure 3.3 – A Simulink model for studying the motor operation

3.5 Calculation of the motor parameters

The rotational speed is related to R.p.m the following way:

$$\omega_{\text{load}} = \frac{\pi \cdot n_r}{30} \text{ [rad/s]}. \quad (3.18)$$

In no-load operation mode, the rotational speed is determined with expression

$$\omega_{\text{no-load}} = \frac{U_A}{K\Phi} \text{ [rad/s]}. \quad (3.19)$$

Value of $K\Phi$ is calculated as:

$$K\Phi = \frac{U_{\text{load}} - I_{\text{load}} R_{A\Sigma}}{\omega_{\text{load}}}, \left[\frac{\text{V} \cdot \text{s}}{\text{rad}} \right], \quad (3.20)$$

where $U_{\text{load}} = U_A$, $I_{\text{load}} = I_A$.

The rotational speed of the motor under load is decreased by

$$\Delta\omega = \frac{I \cdot R_{A\Sigma}}{K\Phi}, \text{ [rad/s]}. \quad (3.21)$$

Inductance of the armature circuit L_A is calculated with formula:

$$L_A = \alpha \cdot \frac{U_{\text{load}}}{2p \cdot n_r \cdot I_{\text{load}}}, \text{ [H]} \quad (3.22)$$

where α is a coefficient for compensated motors chosen from the range of $2 \div 3$;

$$I_{\text{load}} = I_A,$$

Electromagnetic time constant of the armature circuit:

$$T_A = \frac{L_A}{R_{A\Sigma}}, \text{ [s]} \quad (3.23)$$

Electromechanical time constant:

$$T_M = \frac{J \cdot R_{A\Sigma}}{(K\Phi)^2}, \text{ [s]} \quad (3.24)$$

Transfer factor over the motor armature voltage:

$$k_M = \frac{1}{K\Phi}. \quad (3.25)$$

Transfer factor over the motor static torque:

$$k_S = \frac{R_{A\Sigma}}{(K\Phi)^2}. \quad (3.26)$$

To determine the type of transient, damping factor ξ is calculated:

$$W(S) = \frac{k_M}{T^2 S^2 + 2\xi T S + 1} = \frac{k_M}{T_A \cdot T_M \cdot S^2 + T_M \cdot S + 1}.$$

$$2\xi T = T_M,$$

$$T^2 = T_A \cdot T_M,$$

$$\text{therefore, } T = \sqrt{T_A \cdot T_M} \text{ and } \xi = \frac{T_M}{2 \cdot \sqrt{T_A \cdot T_M}}.$$

3.6 Assignment

In accordance with the variant given in table 3.1:

1 – Calculate parameters of a separately excited dc motor.

2 – Build a Simulink model and study the motor operation in the idle mode, $I_s = 0$, and under load, $I_s = I_{load}$. Plot the corresponding graphs.

Study

2.1 – transients of the motor rotational speed and the armature current under no load according to the procedure:

switching-on of the motor → steady-state operation → switching-off of the motor;

2.2 – transients of the motor rotational speed and the armature current under load applied to the motor at zero time, $t=t_0$, simultaneously with the motor starting according to the procedure:

switching-on of the motor and switching-on of the load → steady-state operation → switching-off of the load and switching-off of the motor ;

2.3 – transients of the motor rotational speed and the armature current under load applied and disconnected when the motor operates in the steady-state mode according to the procedure:

switching-on of the motor → steady-state operation → : switching-on of the load → steady-state operation → switching-off of the load → steady-state operation → switching-off of the motor.

3 – Build the block diagram of the motor and obtain the complete differential equation describing the motor behavior in the transient mode and static equation describing the motor behavior in the steady-state mode.

Calculate the steady-state value of the controlled parameter in no-load mode and under load.

Plot a family of static characteristic for several disturbance signals: $I_s = 0.25 * I_{load}$; $I_s = 0.5 * I_{load}$; $I_s = 0.75 * I_{load}$; $I_s = I_{load}$.

4 – Plot time and frequency characteristics of the motor.

5 – Estimate stability with Hurwitz, Mikhailov, Nyquist, and Bode criteria.

6 – Assess the steady-state errors.

7. Assess the time domain specifications of the motor studied.

Table 3.1. Variants of initial data

No variant	Motor type	P_{rated} , kW	n_r , rpm	I_{load} , A	R_A+R_{AP} , Ω	J , kg·m ²	Φ , mWb	$2p$
1	P42	1.5	750	9.75	2.92	0.18	5.1	4
2	P42	2.2	1000	13.3	1.75	0.18	5.2	4
3	P42	4.5	1500	25.4	0.78	0.18	5.1	4
4	P52	3.2	750	19.0	1.078	0.40	7.7	4
5	P52	4.5	1000	25.2	0.632	0.40	7.9	4
6	P52	8.0	1500	43.5	0.259	0.40	8.2	4
7	P62	6.0	750	33.5	0.531	0.65	10.5	4
8	P62	8.0	1000	43.0	0.328	0.65	10.7	4
9	P62	14.0	1500	73.5	0.1275	0.65	11.1	4
10	P71	7.0	750	42.0	0.546	1.4	9.2	4
11	P71	10.0	1000	63.0	0.300	1.4	9.7	4
12	P71	19.0	1500	103.0	0.1235	1.4	10.1	4

Note. Plotting time and frequency characteristics in Matlab Command Window:

$K=$ (value of k_M);

$a_0=$ (product of values T_A*T_M);

$a_1=$ (value of T_M);

$a_2=1$;

Time responses:

$q=$ tf([K],[a0 a1 1])

step(q),grid

impulse(q),grid

Frequency characteristics

$w=0:0.1:1000$;

$p=j*w$;

$q=K./(a_0.*p.^2+a_1.*p+a_2)$

plot(real(q),imag(q));grid

plot(w,imag(q));grid

plot(w,real(q));grid

plot(w,abs(q));grid

bode([K],[a0 a1 a2])

Appendix

Transfer function of a multi-loop control system

Transfer function of a single-loop control system with **negative feedback** is found with the following formula:

$$W(s)_{closed} = \frac{Y(s)}{X(s)} = \frac{W_{forward}(s)}{1 + W_{forward}(s) \cdot W_{feedback}(s)} \quad (3.27)$$

where $W_{forward}(s)$ is the transfer function of the forward path (fig.3.4);

$W_{feedback}(s)$ is the transfer function of the feedback.

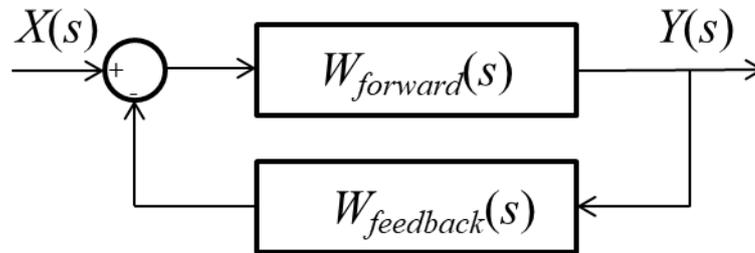


Figure 3.4a – Block diagram of a single-loop control system with **negative** feedback

Transfer function of a single-loop control system with **positive feedback** is found with the following formula:

$$W(s)_{closed} = \frac{Y(s)}{X(s)} = \frac{W_{forward}(s)}{1 - W_{forward}(s) \cdot W_{feedback}(s)} \quad (3.28)$$

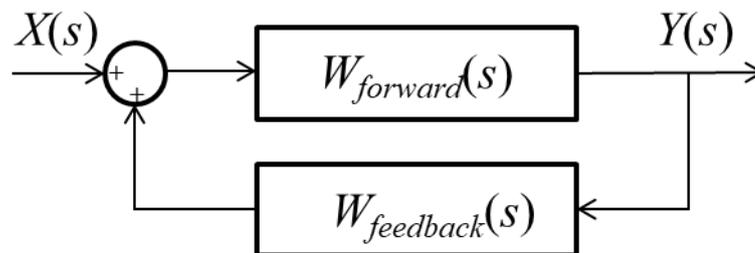


Figure 3.4b – Block diagram of a single-loop control system with **positive** feedback

Before calculating the transfer function of a multi-loop control system, it is necessary to reduce it to a single-loop control system with application of block diagram reduction rules:

Block diagram reduction rules

Rule 1 – Check for the dynamic elements connected in local feedback loop and simplify.

The transfer function of a local feedback connection is found with (3.27).

Rule 2 – Check for the dynamic elements connected in parallel and simplify.

The transfer function of a block parallel connection (fig.3.5) is found by summing the transfer functions of the dynamic elements connected in parallel:

$$W(s)_{parallel} = \sum_{i=1}^k W(s)_i, \tag{3.29}$$

where k is number of dynamic elements connected in parallel.

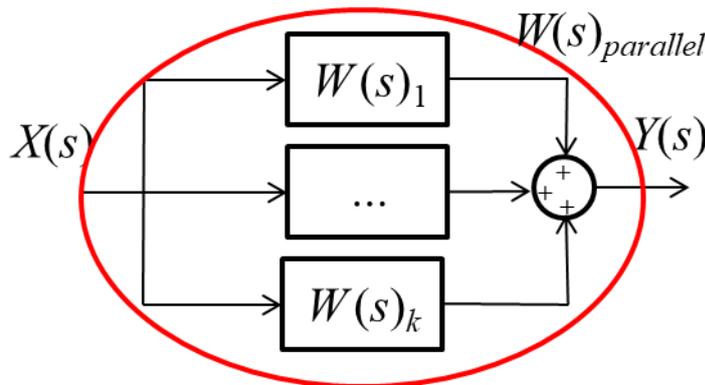


Figure 3.5 – Block diagram of parallel-connected blocks

Rule 3 – Check for the blocks connected in series and simplify.

The transfer function of a block series connection (fig.3.6) is found by multiplying the transfer functions of the dynamic elements connected in series:

$$W(s)_{series} = \prod_i^m W(s)_i, \tag{3.30}$$

where m is number of dynamic elements connected in series.

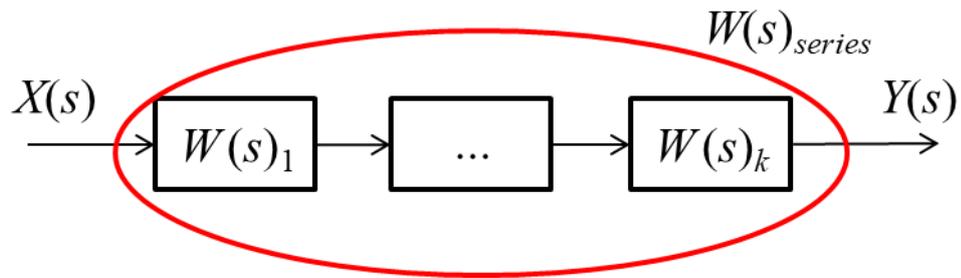


Figure 3.6 – Block diagram of series-connected blocks

Rule 4 – Repeat the above steps until you get the simplified form with one (equivalent) block in the forward path.

Example 1.

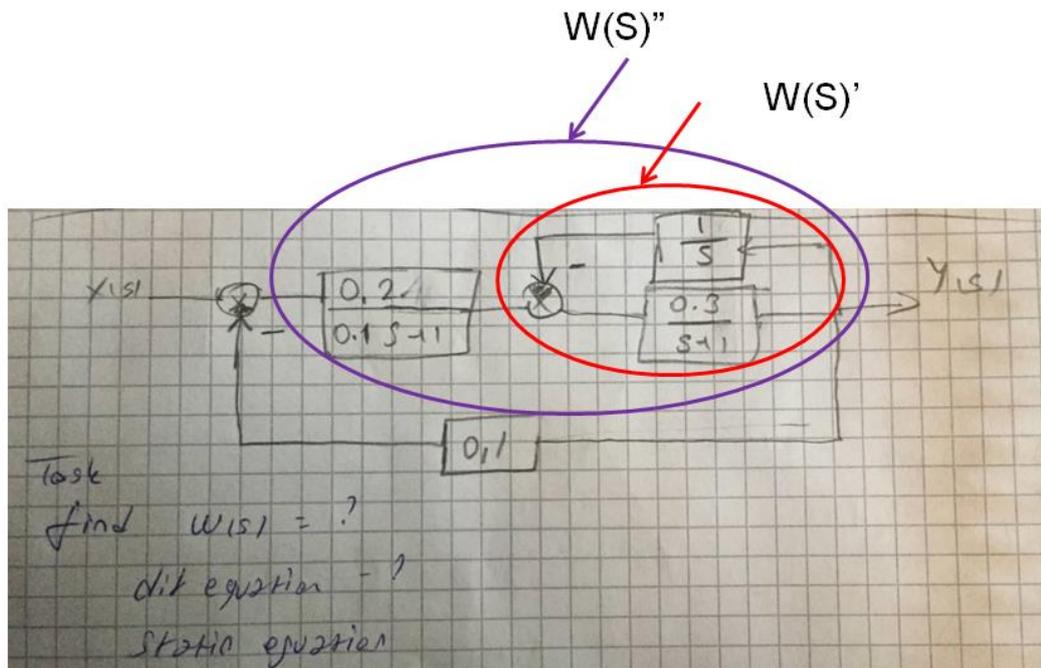


Figure 3.7 – A multi-loop control system

Local feedback connection:

$$\begin{aligned}
 W(s)' &= \frac{W_{l_forward}(s)}{1 + W_{l_forward}(s) \cdot W_{l_feedback}(s)} = \frac{\frac{0.3}{s+1}}{1 + \frac{0.3}{s+1} \cdot \frac{1}{s}} = \frac{\frac{0.3}{s+1}}{\frac{(s+1) \cdot s + 0.3}{(s+1) \cdot s}} = \\
 &= \frac{0.3s}{(s+1) \cdot s + 0.3} = \frac{0.3s}{s^2 + s + 0.3}
 \end{aligned}$$

Series connection:

$$W(s)'' = \frac{0.2}{0.1s+1} \cdot \frac{0.3s}{s^2+s+0.3} = \frac{0.2 \cdot 0.3s}{(0.1s+1) \cdot (s^2+s+0.3)} =$$

$$= \frac{0.06s}{0.1s^3 + s^2 + 0.1s^2 + s + 0.03s + 0.3} = \frac{0.06s}{0.1s^3 + 1.1s^2 + 1.03s + 0.3}$$

The transfer function of the considered closed-loop control system:

$$W(s)_{closed} = \frac{W(s)''}{1 + W(s)'' \cdot W_{feedback}(s)} = \frac{\frac{0.06s}{0.1s^3 + 1.1s^2 + 1.03s + 0.3}}{1 + \frac{0.06s}{0.1s^3 + 1.1s^2 + 1.03s + 0.3} \cdot 0.1} =$$

$$= \frac{\frac{0.06s}{0.1s^3 + 1.1s^2 + 1.03s + 0.3}}{\frac{(0.1s^3 + 1.1s^2 + 1.03s + 0.3) + 0.06s \cdot 0.1}{0.1s^3 + 1.1s^2 + 1.03s + 0.3}} = \frac{0.06s}{0.1s^3 + 1.1s^2 + 1.036s + 0.3}$$

$$W(s)_{closed} = \frac{0.06s}{0.1s^3 + 1.1s^2 + 1.036s + 0.3}$$

$$W(s) = \frac{Y(s)}{X(s)} \quad \Rightarrow \quad Y(s) = W(s) \cdot X(s)$$

$$\frac{0.06s}{0.1s^3 + 1.1s^2 + 1.036s + 0.3} = \frac{Y(s)}{X(s)}$$

The differential equation (written in Laplace transforms) that describes the control system behavior in the dynamic mode is

$$(0.1s^3 + 1.1s^2 + 1.036s + 0.3) \cdot Y(s) = 0.06s \cdot X(s)$$

After opening the brackets

$$0.1s^3 \cdot Y(s) + 1.1s^2 \cdot Y(s) + 1.036s \cdot Y(s) + 0.3 \cdot Y(s) = 0.06s \cdot X(s)$$

The differential equation that describes the control system behavior in the dynamic mode written in time domain is

$$0.1 \frac{d^3 y}{dt^3} + 1.1 \frac{d^2 y}{dt^2} + 1.036 \frac{dy}{dt} + 0.3 y(t) = 0.06 \frac{dx}{dt}$$

The static equation that describes the control system behavior in the steady-state mode is obtained by eliminating all the derivatives from the differential equation:

$$0.3y(t) = 0$$

Therefore, **for any reference signal the output** (which is the controlled parameter $y(t)$) of the control system considered in fig. 4 is equal to **zero**.

Hurwitz stability criterion

Hurwitz stability criterion is based on building special determinants of characteristic polynomial (3.31) that are called Hurwitz determinants

$$Q(s) = a_0s^n + a_1s^{n-1} + a_2s^{n-2} + \dots + a_{n-1}s^1 + a_ns^0 \quad (3.31).$$

For control system to be stable, it is necessary and sufficient that all Hurwitz determinants $\Delta_1, \Delta_2, \dots, \Delta_n$ should be positive.

Hurwitz main determinant is built according to the following rules

- 1) elements of the main diagonal are coefficients of characteristic polynomial (3.31) with increasing index starting from a_1 to a_n ;
- 2) the elements of the columns below the main diagonal are coefficients of (3.31) with successively descending indexes; the elements of the columns above the main diagonal are coefficients with successively increasing indexes;
- 3) 0s are taken instead of coefficients of (3.31) with indexes less than 0 and higher than n .

In accordance with these rules, Hurwitz main determinant of n -th order is the following

$$\Delta_n = \begin{vmatrix} a_1 & a_3 & a_5 & \dots & 0 \\ a_0 & a_2 & a_4 & \dots & 0 \\ 0 & a_1 & a_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a_n \end{vmatrix}. \quad (3.32)$$

Hurwitz determinants of lower order are diagonal subdeterminants of Δ_n ,

$$\begin{vmatrix} a_1 & a_3 & a_5 & \dots & 0 & 0 & 0 \\ a_0 & a_2 & a_4 & \dots & 0 & 0 & 0 \\ 0 & a_1 & a_3 & \dots & 0 & 0 & 0 \\ 0 & 0 & a_2 & \dots & 0 & 0 & 0 \\ - & - & - & - & - & - & - \\ 0 & 0 & 0 & \dots & 0 & a_{n-1} & 0 \\ 0 & 0 & 0 & \dots & a_{n-4} & a_{n-2} & a_n \end{vmatrix}$$

For example, for $n = 3$

$$\Delta_1 = a_1; \quad \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}; \quad \Delta_3 = \begin{vmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & 0 \\ 0 & a_1 & a_3 \end{vmatrix}.$$

The last column of Hurwitz main determinant Δ_n containing only one non-zero element, a_n , we can write

$$\Delta_n = a_n \cdot \Delta_{n-1}. \quad (3.33)$$

Therefore, a_n being positive, it is sufficient to calculate $(n-1)$ Hurwitz determinants, $\Delta_1, \Delta_2, \dots, \Delta_{n-1}$. If they are positive, the control system analyzed is stable.

Example 2.

Let us consider a second-order system, dynamic mode of which is described with o.d.e.

$$0.8 \frac{d^2 y}{dt^2} + 1.2 \frac{dy}{dt} + 2y(y) = 4x(t)$$

The characteristic polynomial is $Q(s) = a_0 s^2 + a_1 s + a_2$ or, numerically, $Q(s) = 0.8s^2 + 1.2s + 2$.

Hurwitz main determinant for the system is

$$\Delta_2 = \begin{vmatrix} a_1 & 0 \\ a_0 & a_2 \end{vmatrix} = \begin{vmatrix} 1.2 & 0 \\ 0.8 & 2 \end{vmatrix} = (1.2 \times 2) - (0.8 \times 0) = 2.4 > 0$$

Stability condition: $\Delta_1 = a_1 = 0.8 > 0$; $\Delta_2 = a_2 \cdot \Delta_1 = a_2 \cdot a_1 = 1.2 \times 2 > 0$.

Thus, a second-order system is stable if all terms of its characteristic polynomial are positive.

Example 3.

Let us consider a third-order system, dynamic mode of which is described with o.d.e.

$$0.1 \frac{d^3 y}{dt^3} + 0.8 \frac{d^2 y}{dt^2} + 1.2 \frac{dy}{dt} + 2y(y) = 4x(t),$$

the characteristic polynomial of which is

$$Q(s) = 0.1s^3 + 0.8s^2 + 1.2s + 2$$

$a_0 \quad a_1 \quad a_2 \quad a_3$

Hurwitz main determinant is

$$\Delta_3 = \begin{vmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & 0 \\ 0 & a_1 & a_3 \end{vmatrix} = a_3 \cdot \Delta_2$$

and numerically

$$\Delta_3 = \begin{vmatrix} 0.8 & 2 & 0 \\ 0.1 & 1.2 & 0 \\ 0 & 0.8 & 2 \end{vmatrix}$$

For the system to be stable, the subdeterminants must be positive

$$\Delta_1 = a_1 > 0; \quad \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} = a_1 \cdot a_2 - a_0 \cdot a_3 > 0, \quad \Delta_3 = a_3 \cdot \Delta_2 > 0.$$

$$\Delta_3 = 2 \times \Delta_2 = 2 \times \begin{vmatrix} 0.8 & 2 \\ 0.1 & 1.2 \end{vmatrix} = 2 \times [(0.8 \times 1.2) - (0.1 \times 2)] = 2 \times (0.95 - 0.2) = 1.5$$

All Hurwitz subdeterminants are positive so the third-order system of example 3 is stable.

The Mikhailov stability criterion formulation

For a control system to be stable, it is necessary and sufficient that the Mikhailov plot should start on the real positive semi-axis at point $[a_n, 0]$ and successively cross n quadrants of the complex plane around the origin only counterclockwise never vanishing and go to infinity in the n -th quadrant, when frequency grows from 0 to infinity, n being the order of the characteristic polynomial of the sys-

tem.

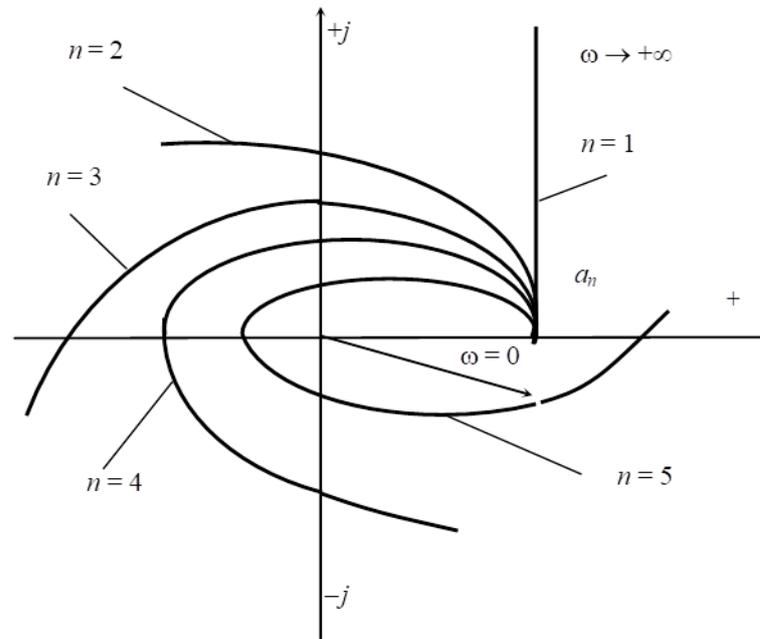


Figure 3.8 – The Mikhailov plots for stable control systems of various order

If the Mikhailov plot crosses zero (the origin of the complex plane), the system is on the stability margin.

If the order of the quadrants that the Mikhailov plot crosses is not successive, the system is unstable.

In Matlab, the Mikhailov plot is built like an ordinary frequency characteristic. In **Matlab Command Window**, the following commands should be written:

```
w=0:0.1:1000;
p=j*w;
q=a0.*p.^n+a1.*p.^(n-1)+...+an;
plot(real(q),imag(q));grid
```

Example 1 Control system 1 is described with the differential equation

$$0.1 \frac{d^3 y}{dt^3} + 0.8 \frac{d^2 y}{dt^2} + 1.2 \frac{dy}{dt} + 2y(y) = 4x(t).$$

The characteristic polynomial of the system is

$$Q(s) = 0.1s^3 + 0.8s^2 + 1.2s + 2.$$

The Mikhailov vector is obtained with $s=j\omega$:

$$D(j\omega) = 0.1(j\omega)^3 + 0.8(j\omega)^2 + 1.2(j\omega) + 2$$

The order of the characteristic polynomial is 3 and $a_n=2$, so if the control system is stable, the Mikhailov plot must start at point $[2, j0]$ at $\omega=0$, go counter-clockwise round origin crossing quadrant 1 and quadrant 2 and go to infinity in quadrant 3 with ω approaching 1000.

In **Matlab Command Window**, we write:

```
w=0:0.1:1000;
```

```
p=j*w;
```

```
q=0.1.*p.^3+0.8.*p.^2+1.2.*p+2;
```

```
plot(real(q),imag(q));grid
```

and obtain the Mikhailov plot in the frequency range from 0 through 1000 1/s.

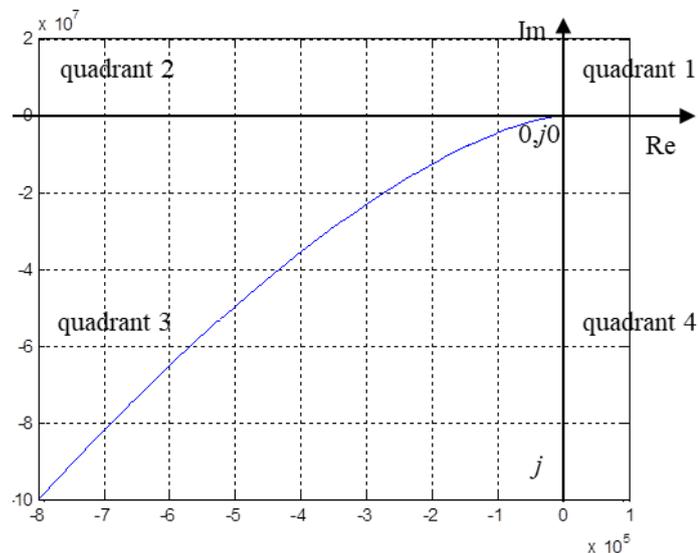


Figure 3.9 – The Mikhailov plot for control system 1

In fig. 3.9, we can see that the Mikhailov plot goes to infinity in quadrant 3 as required for the system stability. However, the scale of the obtained graph is too big to reveal the plot behavior with respect to the origin $0+j0$, that is why it is required to zoom up the domain of the origin.

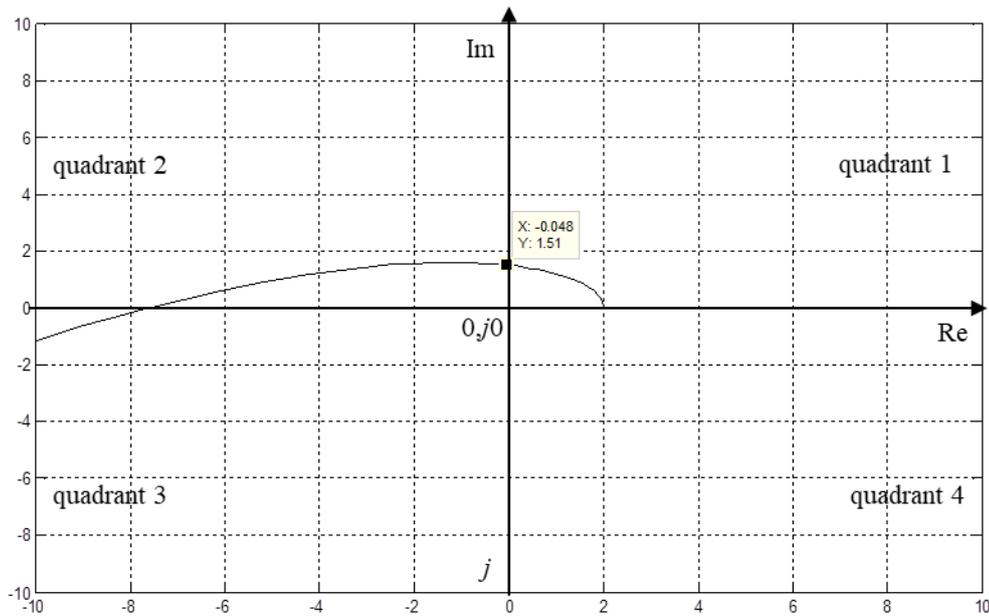


Figure 3.10 –The Mikhailov plot behavior in the domain of the origin

In the zoomed-up presentation of the domain of the origin (fig. 3.10), we can see that the Mikhailov plot starts at the positive real semi-axis at point $[2, j0]$, goes across quadrant 1 counter-clockwise, crosses the positive imaginary semi-axis at point $[0, j1.5]$ and enters quadrant 2, goes across quadrant 2 and enters quadrant 3 at point $[-7.5;0]$.

Therefore, the plot starts at the required point ($[2;j0]$), goes around the origin of the complex plane and successively crosses the quadrants going to infinity in the required quadrant (quadrant 3). It means that the considered control system is stable.

Example 2. A control system is described with the differential equation

$$0.1 \frac{d^3 y}{dt^3} + 0.8 \frac{d^2 y}{dt^2} + 1.2 \frac{dy}{dt} + 12 y(y) = 4x(t).$$

The characteristic polynomial of the system is

$$Q(s) = 0.1s^3 + 0.8s^2 + 1.2s + 12.$$

The Mikhailov vector is obtained with $s=j\omega$:

$$D(j\omega) = 0.1(j\omega)^3 + 0.8(j\omega)^2 + 1.2(j\omega) + 12$$

Control system 2 is also a 3rd-order system, the order of the characteristic polynomial is 3, $a_n=12$. The system is stable if the Mikhailov plot starts on the positive real semi-axis at point $[12, j0]$ at $\omega=0$ and envelops the origin crossing counter-clockwise quadrants 1 and 2 and going to infinity in quadrant 3 with ω approaching 1000.

In **Matlab Command Window**, we write:

```
w=0:0.1:1000;  
p=j*w;  
q=0.1.*p.^3+0.8.*p.^2+1.2.*p+12;  
plot(real(q),imag(q));grid
```

and obtain the Mikhailov plot for system 2 in the frequency range of 0 - 1000 1/s. The zoomed-up presentation of the plot behavior with respect to the origin of the complex plane is given in fig. 3.11.

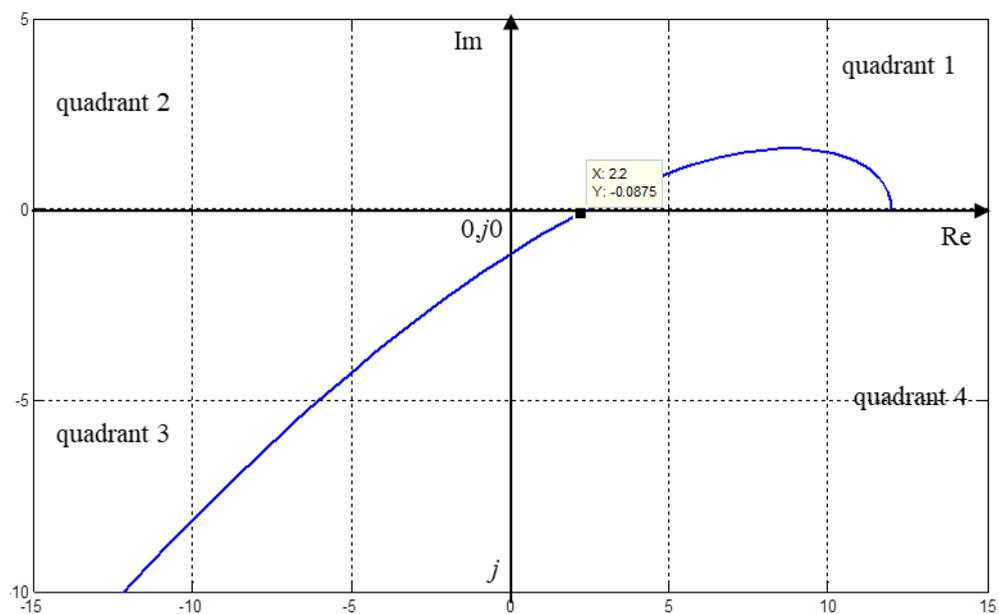


Figure 3.11 – Behavior of the Mikhailov plot for system 2 in the vicinity of the origin

As we can see from the figure, the Mikhailov plot starts at the positive real semi-axis at point $[12, j0]$ (as required), goes counter-clockwise across quadrant 1 (as required) but then crosses the positive Re semi-axis at point $[2.2, j0]$ (instead of crossing the positive Im semi-axis) before enveloping zero and enters quadrant 4, goes across quadrant 4 and cross the negative Im semi-axis at point $[0; -1.2]$ to enter quadrant 3 and go to infinity with the frequency increase.

Such configuration of the Mikhailov plot is typical of unstable control system.

Unit step response $h(t)$ as stability indicator

Unit step signal $1(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ 1, & \text{if } t \geq 0 \end{cases}$ so it is a bounded input. When it is applied to a

stable control system, the system output must be bounded too.

Aperiodic unit step response (fig. 3.12a) shows that the system is quite far from stability margin. Oscillatory unit step response (fig. 3.12b) indicates that the system approaches stability margin. The more oscillations in the transient curve, the closer the system to the stability margin.

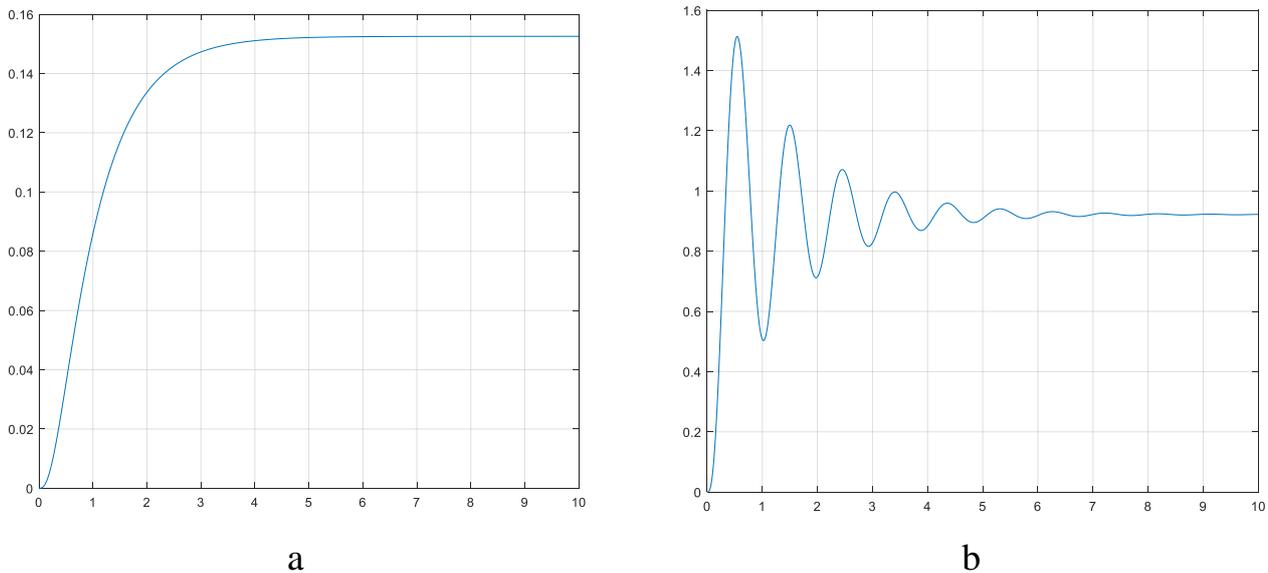


Figure 3.12 – Aperiodic (a) and oscillatory (b) unit step response of a stable control system

Continuous oscillations of the output (fig.3.14) indicate that the system is marginally stable.

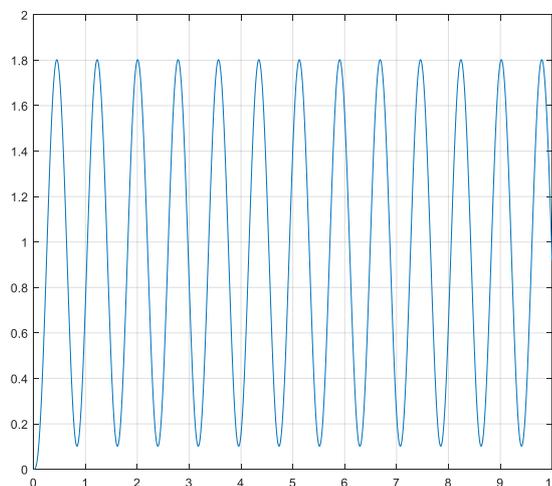


Figure 3.13 – Continuous oscillations of a marginally stable control system output

If the unit-step response goes to infinity in aperiodic (fig.3.14a) or oscillatory (fig.3.14b) manner, the system is unstable.

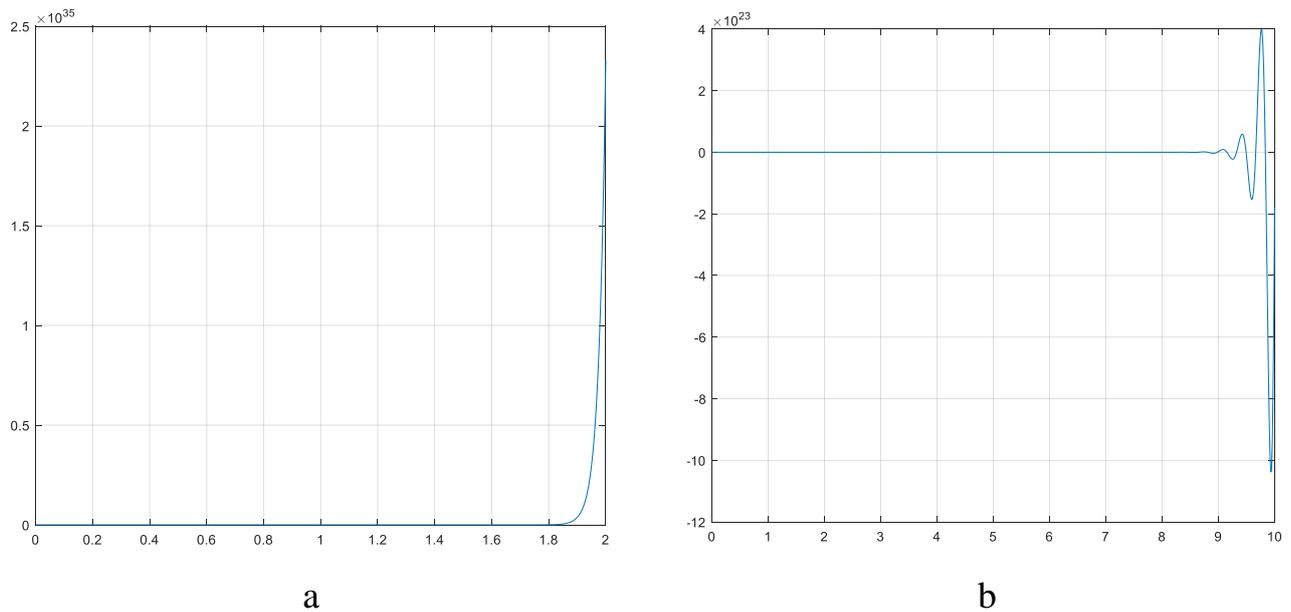


Figure 3.14 – Infinite aperiodic (a) and oscillatory (b) unit-step response of an unstable control system

References to help

1. Transfer Function Of Control System [Electronic resource. Access code: <https://www.electrical4u.com/transfer-function/>]
2. Time Domain Analysis of Control System [Electronic resource. Access code: <https://www.electrical4u.com/time-domain-analysis-of-control-system/>]
3. Time Domain Specifications [Electronic resource. Access code: https://www.tutorialspoint.com/control_systems/control_systems_time_domain_specifications.htm]
4. Steady-State Error: What is is? [Electronic resource. Access code: <https://www.electrical4u.com/steady-state-error-analysis/>]
5. Nyquist Stability Criterion [Electronic resource. Access code: https://www.tutorialspoint.com/control_systems/control_systems_nyquist_plots.htm]
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