Laboratory work 1

Studying of Typical Dynamic Elements of Zero and First Order

1.1 Objective

Study of zero- and first-order typical dynamic elements (TDEs) and their mathematic description.

Building of Simulink models of these dynamic elements and obtaining their time and frequency characteristics.

Gaining command of simulation in Matlab Simulink, plotting time and frequency characteristics, preparing reports on the obtained results in MS Word.

1.2 Mathematical description of dynamic elements (control systems)

Dynamic behavior of any dynamic element or a control system on the whole is described with an ordinary differential equation (o.d.e.), transfer function (T.F.), time and frequency characteristics.

An ordinary differential equation (**dynamic equation**) describes the output signal ***y*(*t*)** behavior in the dynamic (transient) mode. In this mode, the output signal ***y*(*t*)** varies in accordance with its own law. Under action of the reference signal $x(t)$ and disturbance $D(t)$, an o.d.e. is written as:

$$\begin{aligned} a_0\frac{d^n y}{dt^n}+a_1\frac{d^{n-1}y}{dt^{n-1}}+\ldots+a_{n-1}\frac{dy}{dt}+a_n y(t)=b_0\frac{d^m x}{dt^m}+b_1\frac{d^{m-1}x}{dt^{m-1}}+\ldots+b_{n-1}\frac{dx}{dt}+b_m x(t)+\\ +c_0\frac{d^k D}{dt^k}+c_1\frac{d^{k-1}D}{dt^{k-1}}+\ldots+c_{k-1}\frac{dD}{dt}+c_k D(t) \end{aligned}. \quad (1.1)$$

In the steady-state mode, the output signal ***y*(*t*)** varies in accordance with the law specified in the reference (input) signal. In this mode, ***y*(*t*)** behavior is described with an algebraic equation (**static equation**):

$$y(t)=\frac{b_m}{a_n}x(t)+\frac{c_k}{a_n}D(t). \quad (1.2)$$

Transfer function is the ratio of the Laplace transform of the output signal of a dynamic element (control system) to the Laplace transform of the input signal of the dynamic element (control system). It is obtained through substituting complex variable S^i of Laplace transformation instead of i-th derivative: $S=\frac{d}{dt}$; $S^2=\frac{d^2}{dt^2}$, …, $S^n=\frac{d^n}{dt^n}$ under zero initial conditions:

$$(a_0S^n + a_1S^{n-1} + ... + a_{n-1}S + a_n)Y(S) = (b_0S^m + b_1S^{m-1} + ... + b_{n-1}S + b_m)X(S) + \\ +(c_0S^k + c_1S^{k-1} + ... + c_{k-1}S + c_k)D(S) . \quad (1.3)$$

For the reference signal, the transfer function is

$$W(S)_x = \frac{Y(S)}{X(S)} = \frac{b_0S^m + b_1S^{m-1} + ... + b_{m-1}S + b_m}{a_0S^n + a_1S^{n-1} + ... + a_{n-1}S + a_n} . \quad (1.4a)$$

For the disturbance,

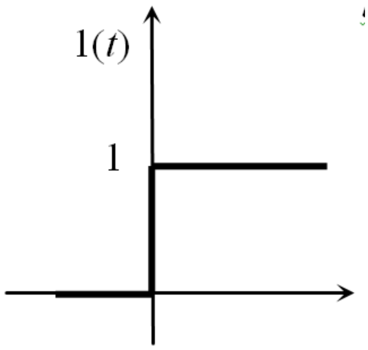
$$W(S)_D = \frac{Y(S)}{D(S)} = \frac{c_0S^k + c_1S^{k-1} + ... + c_{k-1}S + c_k}{a_0S^n + a_1S^{n-1} + ... + a_{n-1}S + a_n} \quad (1.4b)$$

Time characteristics

Time characteristics (responses) specify dynamic behavior of dynamic elements (control systems) in transient modes.

Time responses show a dynamic element or a system response to typical signals under zero initial conditions. There are two time characteristics: unit step response *h*(*t*) and unit impulse response *w*(*t*).

Unit step response (Heaviside response) or transient characteristic *h*(*t*) is the dynamic element (a control system) response to unit step input (Heaviside step function):



$$1(t) = \begin{cases} 0, \text{ if } \ t \le 0 \\ 1, \text{ if } \ t \ge 0 \end{cases} \quad (1.5)$$

Expression for ***h*(*t*)** can be found with the following formula

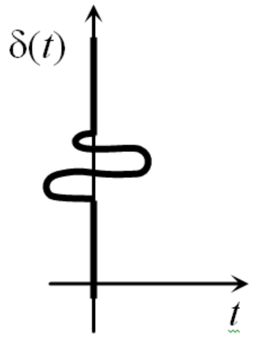
$$h(t) = \frac{R(0)}{Q(0)} + \sum \left(\frac{R(S_i)}{S_i \cdot Q'(S_i)} \cdot e^{S_i t} \right) , \quad (1.6)$$

where *R*(0) and *Q*(0) are, correspondingly, values of the numerator and denominator of the transfer function *W*(*S*) at *S*=0;

S_i is the *i*-th root of the characteristic equation $Q(S) = a_0S^n + a_1S^{n-1} + ... + a_{n-1}S + a_n = 0$;

$R(S_i)$ and $Q'(S_i)$ are, correspondingly, values of the numerator and 1st derivative of the denominator of the transfer function $W(S)$ at $S=S_i$.

Impulse response (Dirac response) or weight function *w*(*t*) is the dynamic element (the control system) response to unit impulse input (Dirac delta function):



$$\delta(t)=\begin{cases}0, & \text{if } t\neq 0\\ \infty, & \text{if } t=0\end{cases} \tag{1.7}$$

Dirac delta function is a unit impulse of an infinitely short duration, infinitely big magnitude, and unity square:

$$\int_{-\infty}^{\infty}\delta(t)dt=1 \tag{1.8}$$

Dirac delta function can be specified as the derivative of Heaviside step function $\delta(t)=\frac{d1(t)}{dt}$, and therefore, Dirac response (weight function) can be determined as the derivative of Heaviside response (transient characteristic):

$$w(t)=\frac{dh(t)}{dt} \tag{1.9}$$

Frequency characteristics

Frequency characteristics describe parameters (magnitude and phase) of the harmonic output of a dynamic element (control system) $y(t)=A_{out}(\omega)\cdot\sin(\omega t+\varphi(\omega))$ as function of frequency of harmonic input $x(t)=1\cdot\sin(\omega t)$.

Amplitude phase frequency characteristic is the hodograph of the vector of complex transfer function $W(j\omega)$ of a dynamic element (control system) in the steady-state mode:

$$W(j\omega)=A_{out}(\omega)\cdot e^{j\varphi(\omega)} \tag{1.10a}$$

or

$$W(j\omega)=A_{out}(\omega)\left(\cos\varphi(\omega)+j\sin\varphi(\omega)\right)\cdot \tag{1.10b}$$

Complex transfer function $W(j\omega)$ can be obtained by substituting $(j\omega)$ instead of S into the transfer function $W(S)$ (see expression (1.4a)):

$$W(j\omega) = W(S)\big|_{S=j\omega} = \frac{b_0(j\omega)^m + b_1(j\omega)^{m-1} + \ldots + b_{m-1}(j\omega) + b_m}{a_0(j\omega)^n + a_1(j\omega)^{n-1} + \ldots + a_{n-1}(j\omega) + a_n} \quad (1.11)$$

Expression (1.11) is a complex number and can be reduced to real and imaginary components

$$W(j\omega) = \mathrm{Re}(\omega) + j\,\mathrm{Im}(\omega) \quad (1.12)$$

Amplitude phase frequency characteristic is plotted on the complex plane with *Re* (real) and *Im* (imaginary) axes.

Real frequency characteristic Re(ω) and **imaginary frequency characteristic Im(ω)** are dependences of, correspondingly, real and imaginary components of complex transfer function $W(j\omega)$ on frequency of harmonic input signal

$$\mathrm{Re}(\omega) = A_{out}(\omega) \cdot \cos\big(\varphi(\omega)\big) \quad (1.13a)$$

$$\mathrm{Im}(\omega) = A_{out}(\omega) \cdot \sin\big(\varphi(\omega)\big) \quad (1.13b)$$

Expressions for these characteristics can be found after presenting complex transfer function $W(j\omega)$ (1.11) as a complex number with real and imaginary components.

Amplitude frequency characteristic *A*(ω) is frequency dependence of the complex transfer function modulus:

$$A(\omega) = |W(j\omega)| = \sqrt{\mathrm{Re}^2(\omega) + \mathrm{Im}^2(\omega)} \quad (1.14)$$

Amplitude frequency response shows change in the amplitude ratio of the dynamic element (control system) output and input with frequency.

Phase frequency characteristic φ(ω) is frequency dependence of the complex transfer function argument of a dynamic element (control system) in the steady-state mode:

$$\varphi(\omega) = ArgW(j\omega) = arctg\,\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)} \quad (1.15)$$

Phase frequency response shows change in the phase angle between the dynamic element (control system) output and input with frequency.

Logarithmic characteristic, or **Bode diagram**, comprises two plots, logarithmic magnitude plot in dB and logarithmic phase plot in degrees. In both plots, *x*-axis represents angular frequency (logarithmic scale).

Logarithmic magnitude plot is

$$L(\omega) = 20\lg[A(\omega)] \qquad (1.16)$$

1.3 Typical dynamic elements of zero- and first-order

TDEs of zero- and first-order include proportional (P), integrating (I), differentiating (D), and first-order aperiodic (A-I) dynamic elements.

Mathematical description of these TDEs is given in tables 1.1 and 1.2.

Table 1.1 – O.d.e. and transfer functions of zero- and first-order TDEs

TDE	o.d.e.	Transfer function $W(S)$
P	$y(t)=Kx(t)$	$W(S)_P=K$
I	$T\frac{dy(t)}{dt} = Kx(t) \Rightarrow y(t) = \frac{K}{T}\int x(t)dt$	$W(S)_I = \frac{K/T}{S}$
D	$y(t) = K \cdot T\frac{dx(t)}{dt}$	$W(S)_D = (K \cdot T)S$
A-I	$T\frac{dy(t)}{dt} + y(t) = Kx(t)$	$W(S)_{A-I} = \frac{K}{TS+1}$

Table 1.2–Time responses and complex transfer functions of 0- and 1st-order TDEs

TDE	$h(t)$	$w(t)$	$W(j\omega)$
P	$h(t)_P=K$	$w(t)_P = \begin{cases} 0, \text{ if } t \neq 0 \\ \infty, \text{ if } t = 0 \end{cases}$	$W(j\omega)_P=K+j0$
I	$h(t)_I = (K/T)t$	$w(t)_I = K/T$	$W(j\omega)_I = 0 - j\frac{K/T}{\omega}$
D	$h(t)_D = \begin{cases} 0, \text{ if } t \neq 0 \\ \infty, \text{if } t = 0 \end{cases}$	$w(t)_D = \begin{cases} 0, \text{ if } t \neq 0 \\ \pm\infty, \text{ if } t = 0 \end{cases}$	$W(j\omega)_D = 0 + j(K \cdot T)\omega$
A-I	$h(t)_{A-I} = K\left(1 - e^{-\frac{1}{T}t}\right)$	$w(t)_{A-I} = \frac{K}{T}e^{-\frac{1}{T}t}$	$W(S)_{A-I} = \frac{K}{jT\omega+1}$ or $W(S)_{A-I} = \frac{K}{(T\omega)^2+1} - j\frac{(K \cdot T)\omega}{(T\omega)^2+1}$

1.4. Simulink model for studying typical zero- and first-order dynamic elements

1.4. 1. Building of a Simulink model

A Simulink model is built in “model” window by dragging required blocks from corresponding block libraries in **Simulink Library Browser** to the model window.

Time characteristics are obtained by applying relevant signals to the dynamic element studied. Frequency characteristics are plotted with **MatLab** commands.

The Simulink model for proportional (P), integrating (I), differentiating (D), and first-order aperiodic (A-I) dynamic elements is shown in fig. 1. The model in the figure is given in the general form. In the definite model built by a student, values of transfer coefficient *K* and time constant *T* must be taken according to the variant (table 1.7).

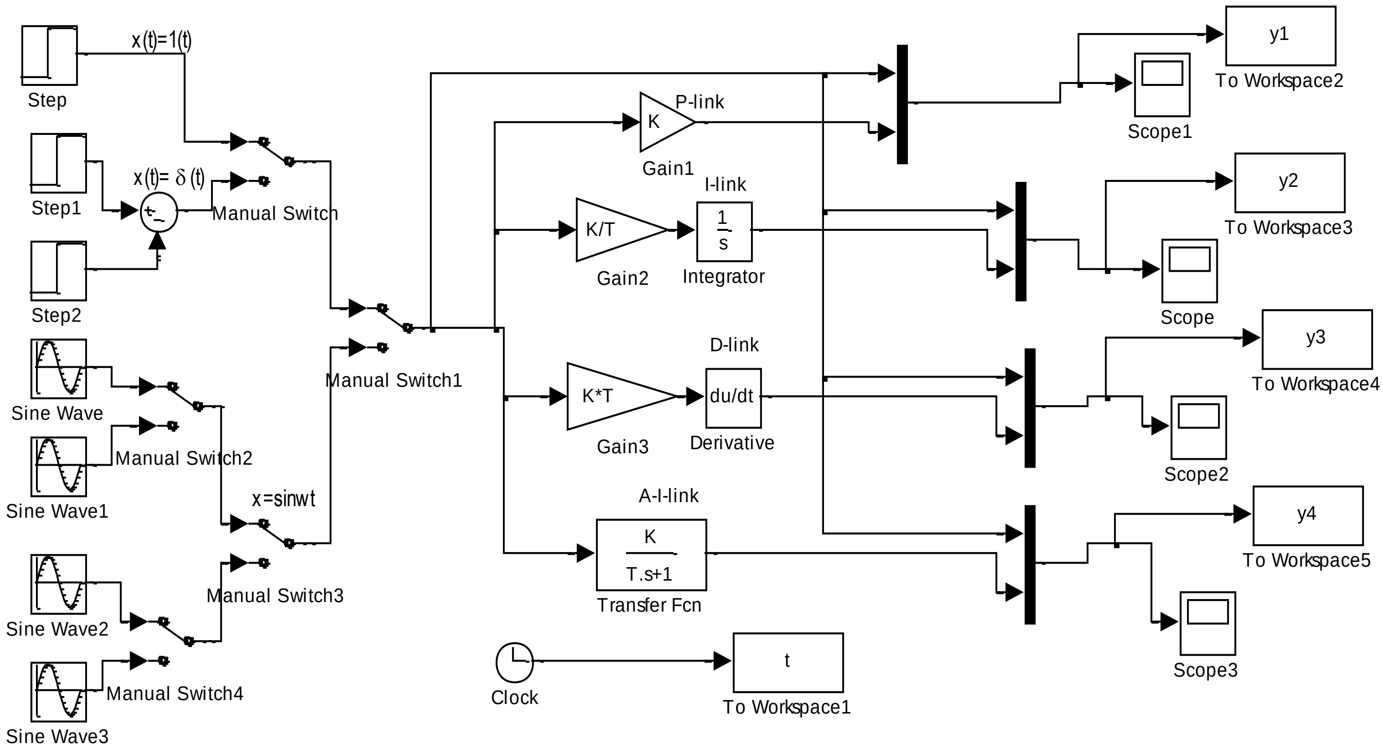


Figure 1.1 – Simulink model for studying typical dynamic elements of zero and first order

1.4 2. Adjustment of the simulation parameters

Before starting simulation, it is necessary to perform general adjustment of the simulation parameters in **Configuration parameters** window (fig. 1.2–1.3) and set the required parameters of the model blocks in **Block parameter** window of every block.

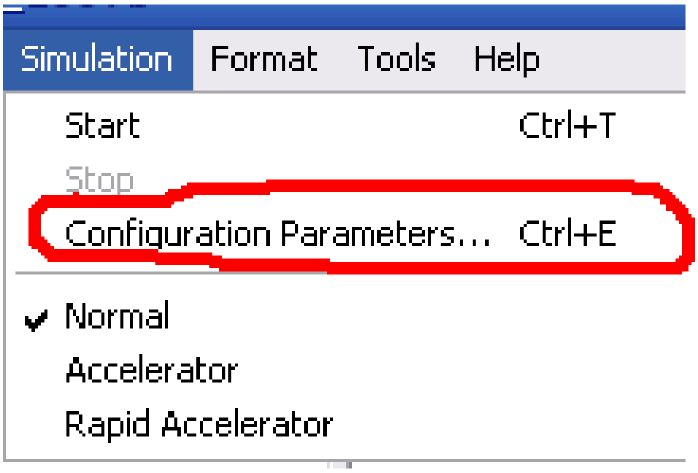


Figure 1.2 – Entry to the simulation parameter adjustment

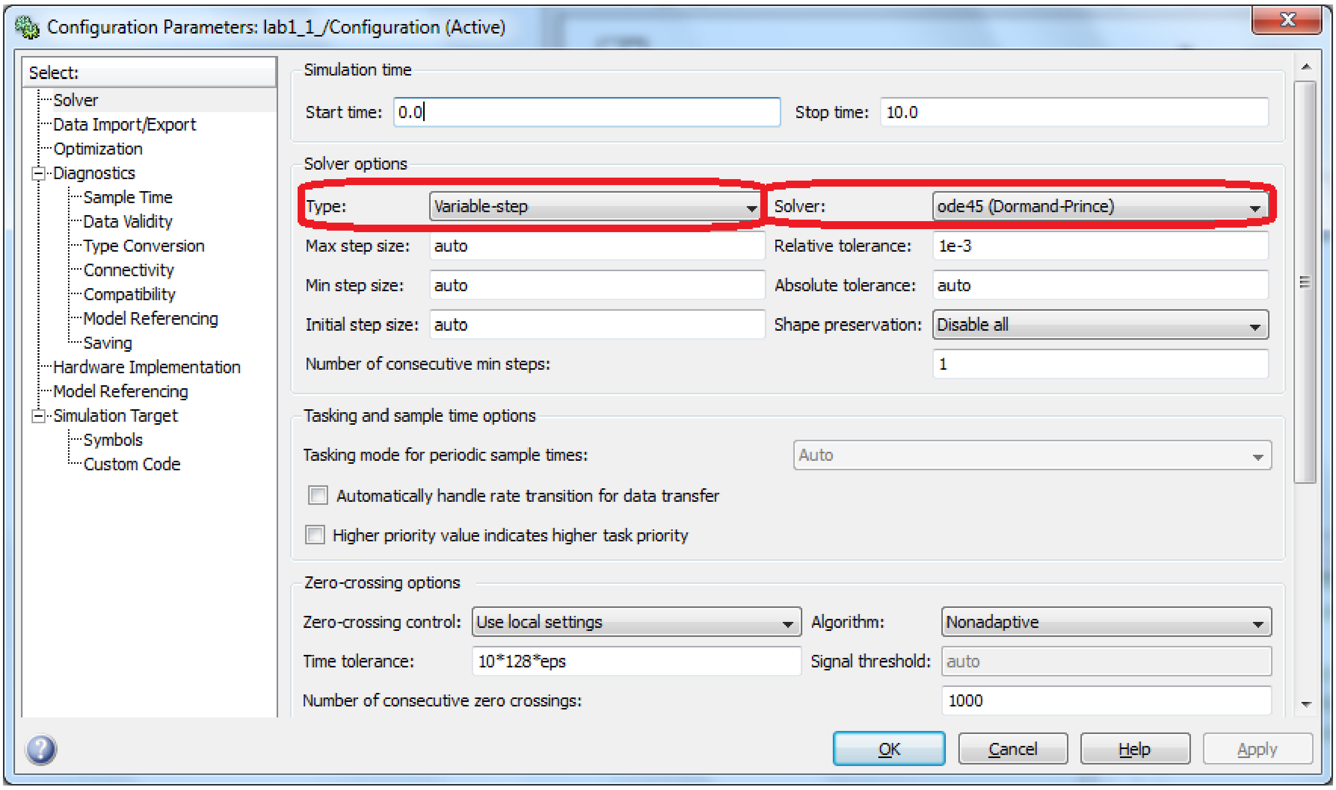


Figure 1.3 - **Configuration parameters** window with default settings

In fig. 1.3, **Start time** field sets the start time of simulation, it is typically 0.

Stop time field sets the stop time of simulation and specifies the simulation duration that must be long enough to show the transient in the studied dynamic element for every input. For P, I, and D elements with no transient, stop time can be **Stop time = 2.** For A-I element, **Stop time** is determined by the end of transient and start of steady-state mode. Usually, it is equal to (3 – 4)T, where T is time constant of the element studied.

For sine input, the stop time is chosen so as to show two or three periods of the simulated signal. For sine frequency ω = 1, stop time t_{end} **= 4*π/ω1** (two periods).

In **Solver** field, a numerical technique for solving differential equations is chosen. The default settings are shown in fig. 1.3.

To obtain the required characteristics, it is necessary to change the settings according to fig. 1.4:

– to select **Fixed-step** from the list in **Type** field;

– to set size of the fixed step *dt1* in **Fixed step size** field not exceeding 0,01*T*;

– to select **ode4 (Runge-Kutta)** from the list in **Solver** field

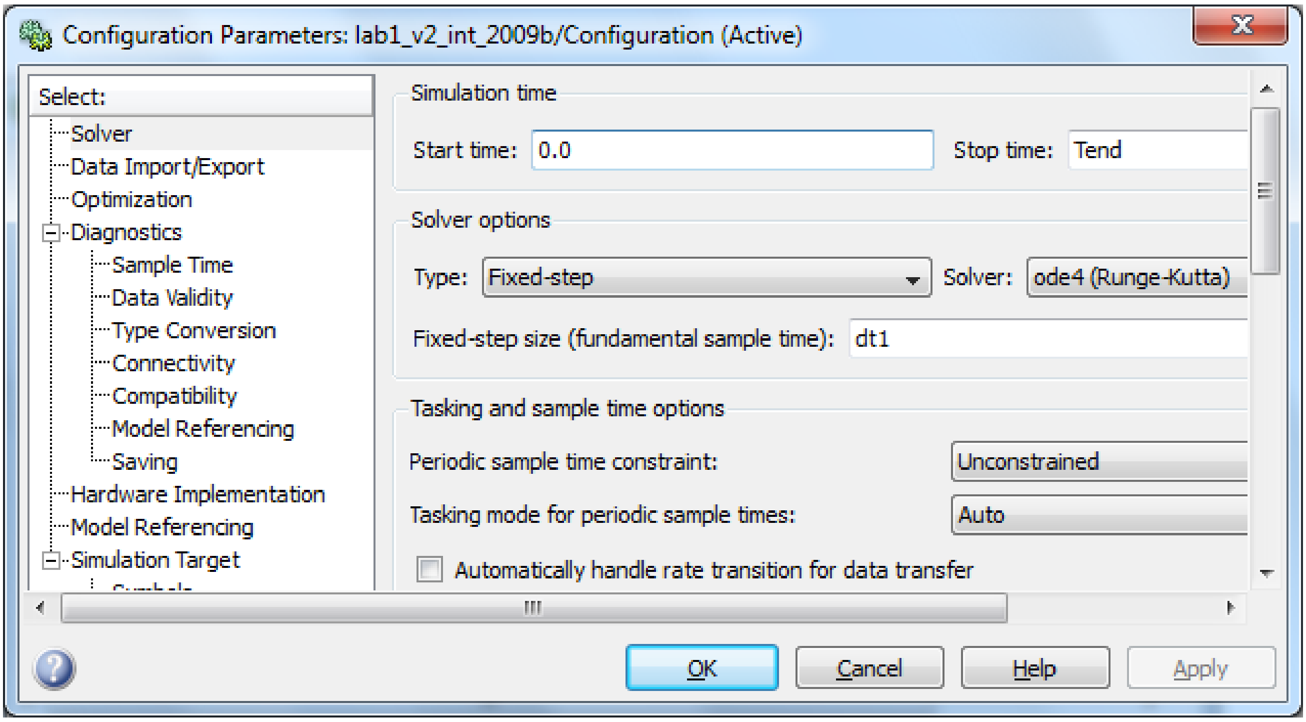


Figure 1.4 - **Configuration parameters** window after configuration parameter adjustment (parameters **Tend** and **dt1** must be set numerically)

1.5 Setting of the model blocks parameters

1.5.1 Blocks for simulating input signals

Blocks for simulating input signals are taken from **Sources** library.

1.5.1.1 **Step** block is used to simulate unit step (Heaviside step function) $\boldsymbol{x(t) = 1(t)}$ and unit impulse (Dirac delta function) $\boldsymbol{x(t) = \delta(t)}$.

The block parameters to set are the following:

Step time field specifies the time of step (default setting is 1)

Initial value field enters the initial value of the step (default setting is 0)

Final value field specifies the final value of the step (default setting is 1).

In the laboratory work, these parameters are set depending on the input signal type.

To obtain step response *h*(*t*), the settings in these fields in **Step** block in the model (see fig. 1.1) must be changes according to fig. 1.5.

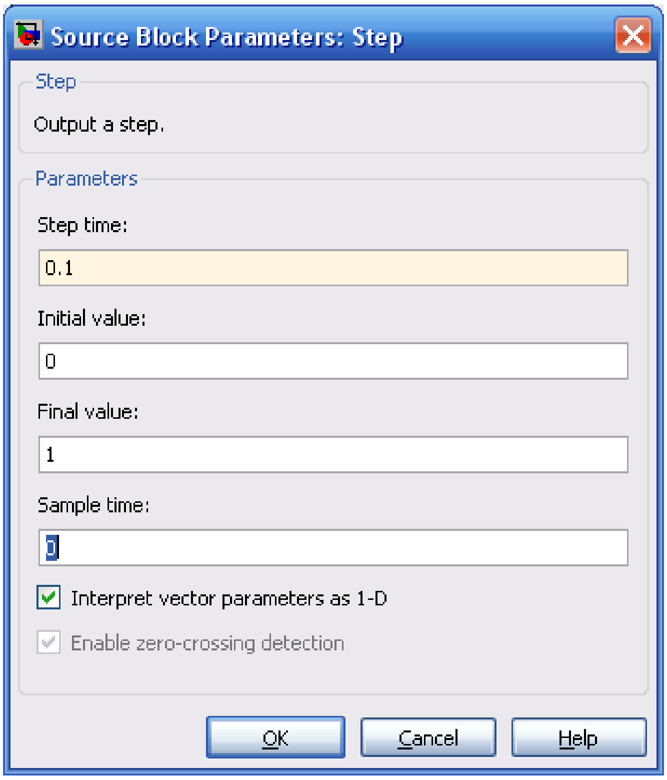


Figure 1.5 – Required settings of **Step** parameters to obtain unit step response

To obtain unit impulse response *w*(*t*), two **Step** blocks are connected with **Sum** block (**Step1** and **Step 2** blocks in fig. 1.1). The parameters of these blocks are given in table 1.3.

Table 1.3 – Parameters of **Step 1** and **Step 2** blocks

Parameter	**Step1** block	**Step2** block
Step time	0.001	0.002
Initial value	0	0
Final value	1000	1000

1.5.1.2 **Sine Wave** block is used to simulate harmonic input $\boldsymbol{x(t) = A_{in}\sin\omega t}$.

The block parameters to set are the following (fig. 1.7):

Amplitude field specified the sine wave magnitude A_{in} (default setting is **1**).

Frequency field specifies the sine wave frequency ω (default setting is **1**).

In the laboratory work, the magnitude of the harmonic input A_{in} = **1** so **Amplitude** field remains unchanged in all four blocks **Sine Wave** used in the model (see fig. 1.1), while **Frequency** field changes according to the variant values of sine wave frequency given in table 1.7.

Other fields in **Source Block Parameters: Sine Wave** window remain with default settings.

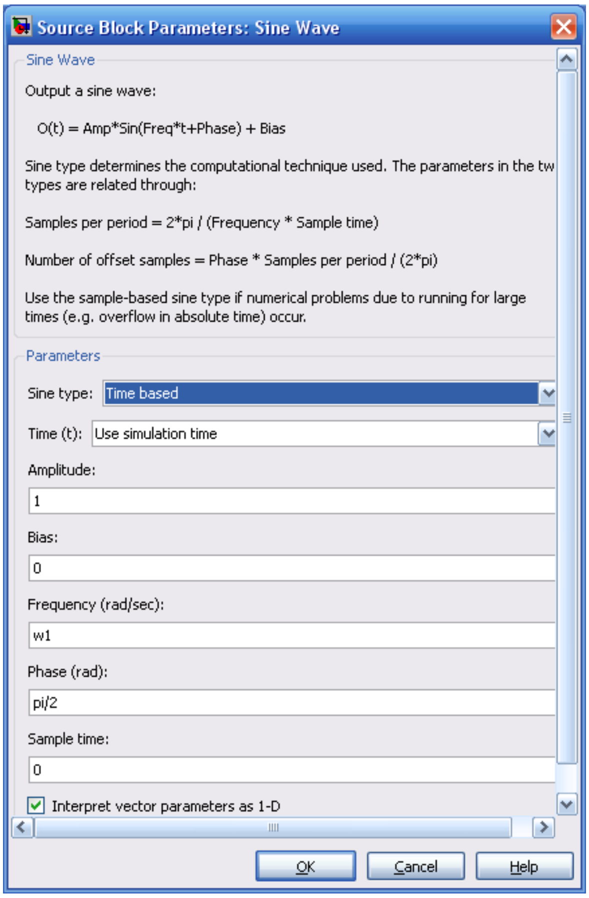


Figure 1.6 - **Source Block Parameters: Sine Wave** window (parameter **w1** must be set numerically for each **Sine Wave** block according to frequency specified in the variant)

1.5.2 Blocks for visualization of output signals

Blocks for visualization of output signals are taken from **Sinks** library.

1.5.2.1 **Scope** block represents an oscillograph. Double click on the block opens the oscillograph screen window. To view the signals plotted, it may be necessary to press “binoculars” icon.

1.5.2.2 **To Workspace** block is used to withdraw simulation results from **Simulink** to **Matlab** to process them. The setting of the block parameters is shown in fig. 1.7.

In **Variable name** field, the name of the variable withdrawn is entered. Letter *y* designates output variable, letter *t* – time.

In **Save format** field, **Array** must be selected from the list.

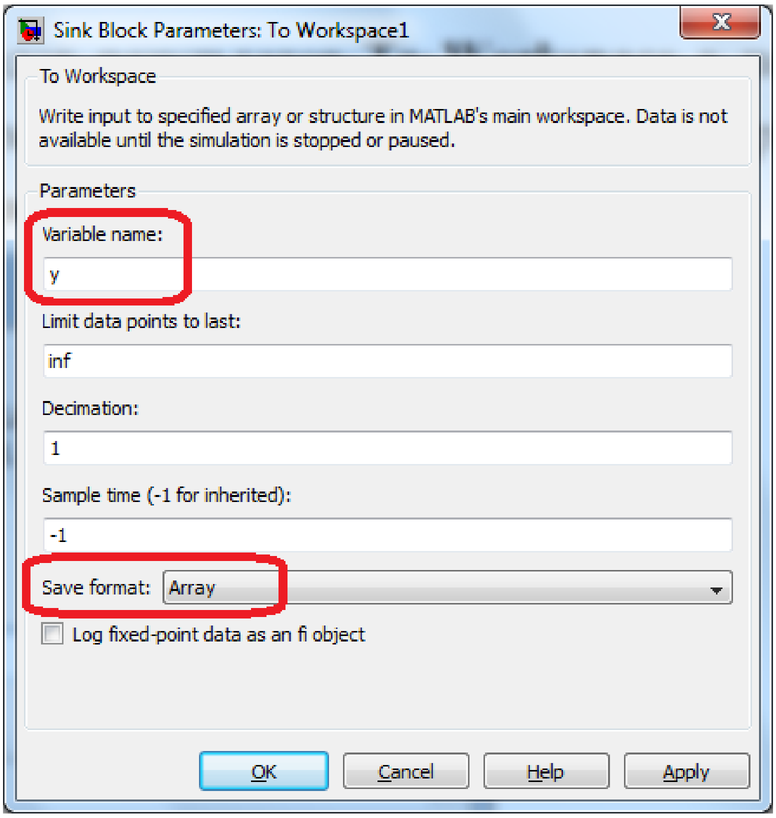


Figure 1.7 – Settings of **To Workspace** block parameters

1.5.3 Blocks for simulating TDE

1.5.3.1 **Gain** block located in **Math Operations** library represents a numerical coefficient.

1.5.3.2 **Integrator** and **Derivative** blocks execute, correspondingly, integration and differentiation of an input signal. These blocks are located in **Continuous** library and have no parameters to adjust (in this laboratory work). A numerical coefficient is added with **Gain** block connected in series.

1.5.3.3 **Transfer Fcn** block

Transfer Fcn block parameters are set by entering corresponding values of polynomial coefficients in the numerator and denominator of the transfer function simulated, the values separated with a space. In the square brackets on the right, the coefficient at S^0 is placed, the order of *S* increasing from the right to the left.

Figure 1.8 presents an example of setting block **Transfer Fcn** parameters for a transfer function of an A-I typical dynamic element

$$W(s) = \frac{2}{0.1s + 1} \text{ or } W(s) = \frac{2s^0}{0.1s^1 + 1s^0}$$

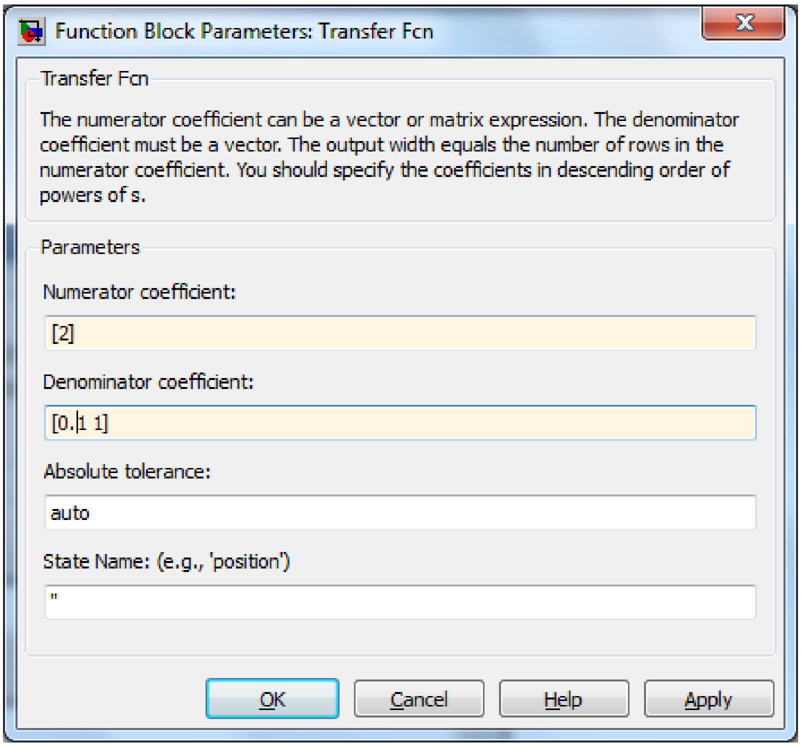


Figure 1.8 – Setting of block **Transfer Fcn** parameters for the considered example

1.6. Simulation and processing of simulation results

1.6.1 Simulation start

The simulation is launched with command **Simulation→Start simulation** or by clicking on **Start simulation** icon (black triangular) in the model window (fig. 1.9). It is also possible to enter command **sim(MyFile)** in **Matlab Command Window**, where **MyFile** is the name of the model file, for example **sim('lab1_5_int_2009b.mdl')**.

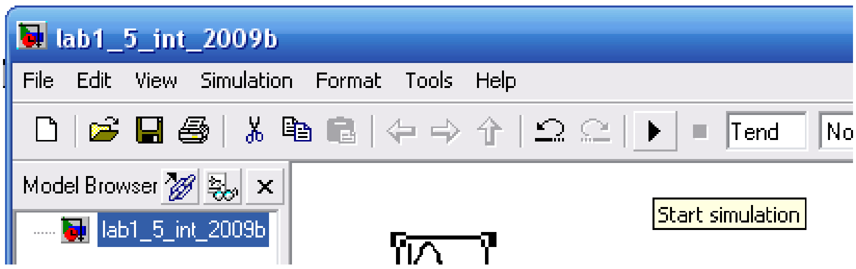


Figure 1.9 – Simulation start

After simulation stops, open block **Scope** window with a left double click to see the graphs. In case the curves obtained are distorted, the configuration parameters must be corrected (for example, fixed step size should be made smaller) and the simulation repeated.

1.6.2 Simulation graph plotting

To plot graphs of the input and output signal simulated, command **plot(t,y*i*),grid**, where *i* is 1 – for P-TDE, 2 – for I-TDE, 3 – for D-TDE, and 4 – for A-I-TDE, must be entered in **Matlab Command Window**.

In **Figure** window that opens, the graphs must be edited with the help of command **Edit** of the top menu. It is necessary:

– to set relevant axes limits to display the graphs distinctly with command **Edit→Axes Properties→Limits**;

–to click on each graph and edit the graph line with command **Edit→Current Object Properties→Style**: **Solid line** for the output signal and **Dashed line** for the input signal; in **Line width** field to select **2.0**, and in **Line color** to select **Black** for both lines.

1.6.3 Building of frequency characteristics

Frequency characteristics are built with application of the transfer functions of the typical dynamic elements studied. Table 1.4 presents commands that are entered in **Matlab Command Window** to obtain frequency characteristics.

Table 1.4 – Commands entered in **Matlab Command Window**

Frequency characteristics	Commands
Amplitude-phase frequency characteristic $W(j\omega)$	**plot(real(q),imag(q)),grid**
Real frequency characteristic $\mathrm{Re}(\omega)$	**plot(w,real(q)),grid**
Imaginary frequency characteristic $\mathrm{Im}(\omega)$	**plot(w,imag(q)),grid**
Amplitude frequency characteristic $A(\omega)$	**plot(w,abs(q)),grid**
Bode magnitude and phase plot $L(\omega)$, $\varphi(\omega)$	**bode([K],[a0 a1])**

Table 1.5 presents <u>examples of commands</u> in **Matlab Command Window** for obtaining frequency characteristics of each TDE studied in the work with transfer coefficient K=2 and time constant T=0.1.

Table 1.5 – Examples of commands for obtaining frequency characteristics

Typical dynamic element	Transfer function	Commands entered in **Matlab Command Window**

Proportional P-TDE	$W(s)=2$	w=0:0.1:200;
		p=j*w;
		q=2.;
		plot(real(q),imag(q)),grid
		plot(w,real(q)),grid
		plot(w,imag(q)),grid
		plot(w,abs(q)),grid
		bode([2],[1]),grid

Integrating I-TDE	$W(s)=\dfrac{2}{0.1s}$	w=0.001:0.01:200;
		p=j*w;
		q=2./(0.1.*p+0);
		plot(real(q),imag(q)),grid
		plot(w,real(q)),grid
		plot(w,imag(q)),grid
		plot(w,abs(q)),grid
		bode([2],[0.1 0]),grid

Differentiating D-TDE	$W(s)=2\cdot 0.1s$	w=0:0.1:200;
		p=j*w;
		q=2.*0.1.*p;
		plot(real(q),imag(q)),grid
		plot(w,real(q)),grid
		plot(w,imag(q)),grid
		plot(w,abs(q)),grid
		bode([2*0.1 0],[1]),grid

Aperiodic A-I-TDE	$W(s)=\dfrac{2}{0.1s+1}$	w=0:0.01:2000;
		p=j*w;
		q=2./(0.1.*p+1);
		plot(real(q),imag(q)),grid
		plot(w,real(q)),grid
		plot(w,imag(q)),grid
		plot(w,abs(q)),grid
		bode([2],[0.1 1]),grid
		freqs([2],[0.1 1]),grid;

1.7 Preparation of the report on the laboratory work

The report on the laboratory work must be prepared in **MS Word** document.

The report must contain the **Simulink** model of the typical dynamic elements studied and all the graphs obtained via simulation and built via **Matlab** commands.

Each dynamic element studued must be mathematically described with the corresponding differential and algebraic (if possible) equations and transfer function (numerical expressions with parameters K and T according to the variant). All the graphs (signals) must be described with the appropriate mathematical expression.

The **Simulink** model is copied from **Simulink** with command **Edit→Copy model to clipboard** in the model window and pasted into the to the **MS Word** document with **Edit→Paste** command.

The graphs are copied with command **Edit→Copy Figure** in the **Matlab Figure** window and pasted into the **MS Word** document with **Edit→Paste** command.It is also possible to save figures as files to further insert them as pictures from the files.

For every TDE, the preferable arrangement of the graphs in the report studied is shown in table. 1.6

Table 1.6 – Arrangement of the graphs for each dynamic element in the report

Name of the typical dynamic link	
Mathematical description	
Unit step response	Unit impulse response
Sine wave 1 response	Sine wave 2 response
Sine wave 3 response	Sine wave 4 response
Amplitude-phase frequency characteristic $W(j\omega)$	Real frequency characteristic $\mathrm{Re}(\omega)$
Imaginary frequency characteristic $\mathrm{Im}(\omega)$	Amplitude frequency characteristic $A(\omega)$
Bode magnitude and phase plot $L(\omega)$, $\varphi(\omega)$	

1.8 Assignment

Build a Simulink model (see fig. 1.1) for studying zero- and first-order typical dynamic elements with the numerical parameters specified in the variant given in table 1.7.

Make simulation and obtain TDEs time and frequency characteristics.

Describe with studied dynamic elements and their characteristics numerically with the help of formulae (1.11)-(1.16) and expressions given in tables 1.1-1.2.

Identify the signals (which is the input and which is the output) in the figures with two curves.

Table 1.7 – Variants of the initial data

№	***K***	***T***	**ω1**	**ω2**	**ω3**	**ω4**
1.	2	0,04	1	5	15	90
2.	1,5	0,3	1	10	40	100
3.	0,5	0,01	1	4	20	80
4.	3	0,06	1	2	10	75
5.	0,5	0,02	1	15	75	150
6.	2,5	0,05	1	5	30	120
7.	0,25	0,5	1	3	15	90
8.	4	0,2	1	20	50	100
9.	0,75	0,01	1	5	25	150
10.	3,5	0,7	1	6	30	120

Appendix
Basic characteristics of zero- and first-order TDEs in general form

<u>Proportional TDE</u>

Dynamic equation $y(t)=kx(t)$

Transfer function $W(S)=k$

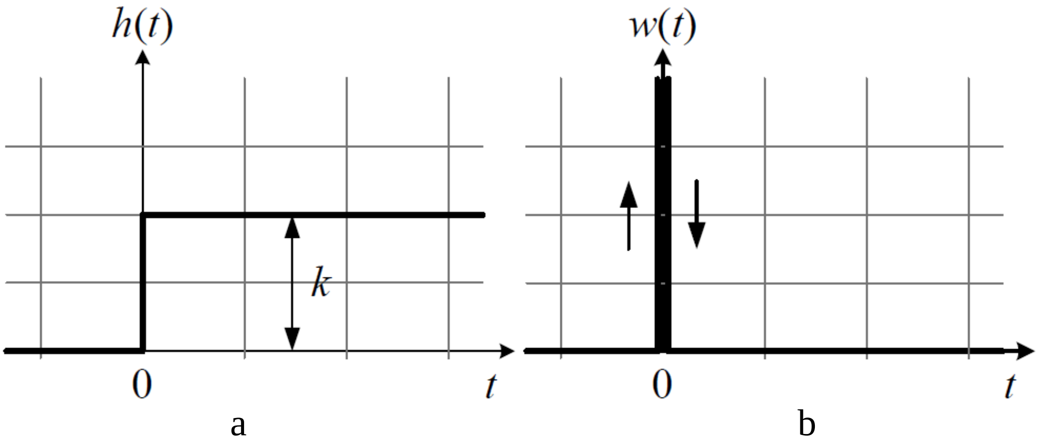


Figure 1.10 – Time characteristics of P-TDE

a – unit step response (transient function) $h(t)=k\cdot 1(t)$,

b – unit impulse response (weight function) $w(t)=\dfrac{dh(t)}{dt}=k\cdot\delta(t)$

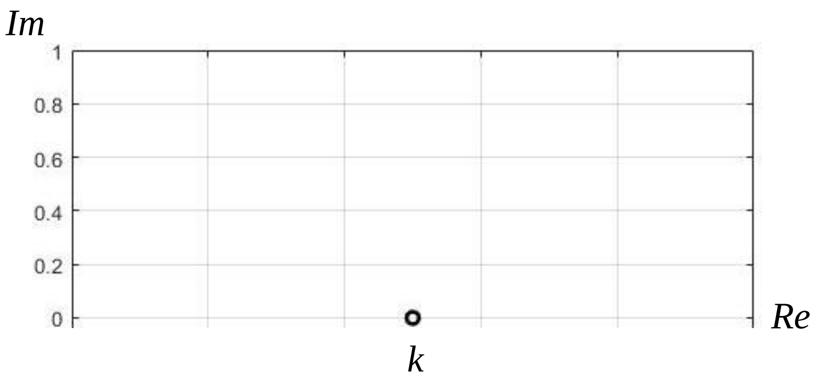


Figure 1.11 – Amplitude-phase characteristic of P-TDE

$W(j\omega)=A(\omega)e^{j\varphi(\omega)}=k\cdot e^{j0}$ or $W(j\omega)=k+j0$

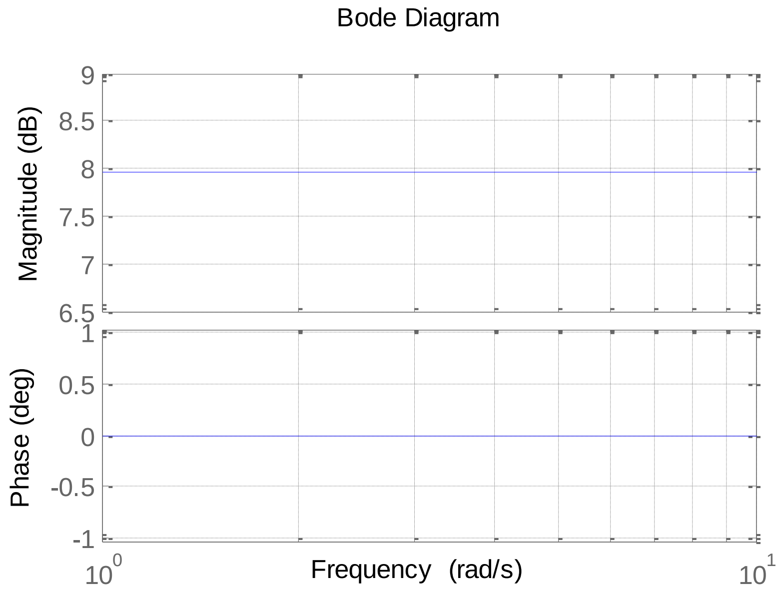
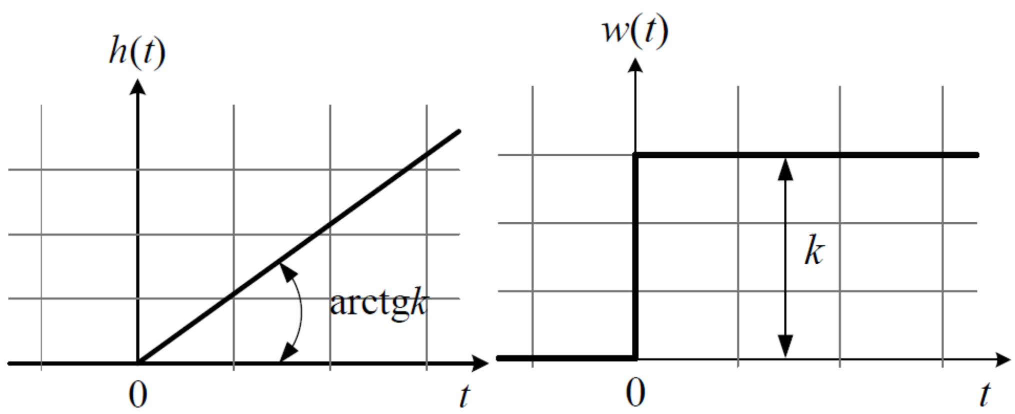


Figure 1.12 – Bode plot of P-TDE:
bode magnitude plot $L(\omega)=20\lg A(\omega)=20\lg k$;
bode phase plot $\varphi(\omega)=0)$

Integrating TDE

Dynamic equation $\frac{dy}{dt}=kx(t)$

Transfer function $W(S)=\frac{k}{S}$



a b

Figure 1.13 – Time characteristics of I-TDE
a – unit step response (transient function) $h(t)=k\cdot t$,
b – unit impulse response (weight function) $w(t)=\frac{dh(t)}{dt}=k$

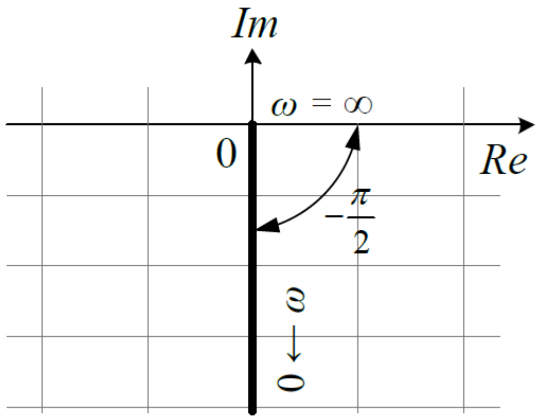


Figure 1.14 – Amplitude-phase characteristic of I-TDE

$$W(j\omega) = A(\omega)e^{j\varphi(\omega)} = \frac{k}{\omega}e^{-j\frac{\pi}{2}} \text{ or } W(j\omega) = 0 - j\frac{k}{\omega}$$

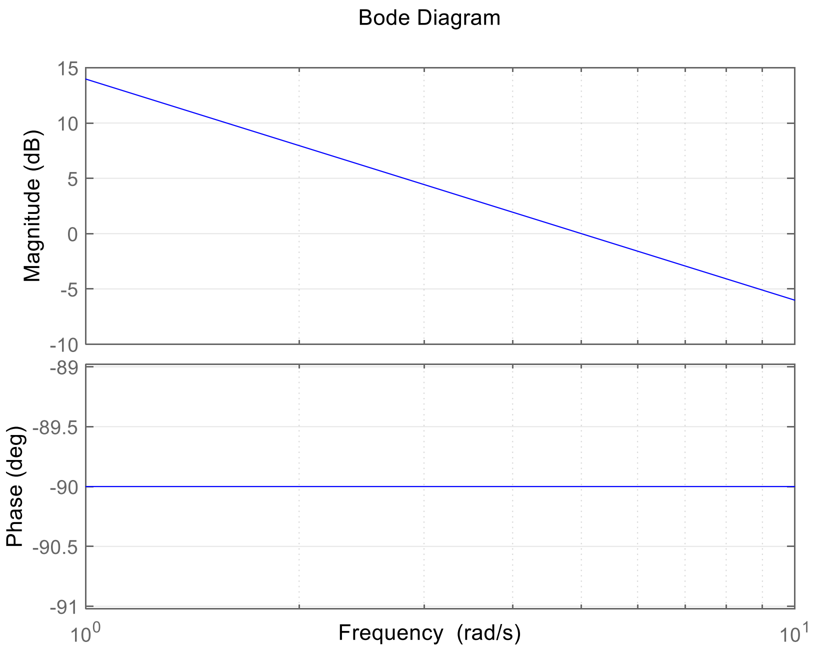
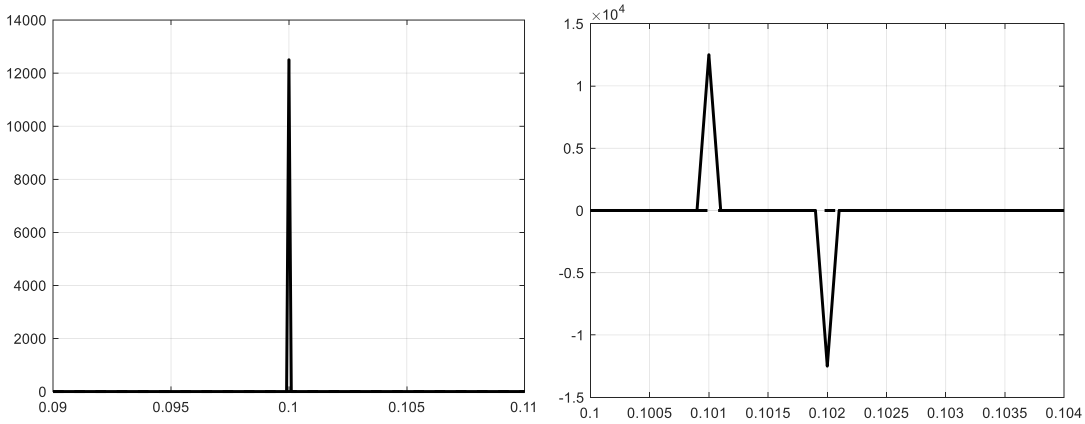


Figure 1.15 – Bode plot of I-TDE:
bode magnitude plot $L(\omega)$=20lg$A(\omega)$ =20lgk/ω;
bode phase plot $\varphi(\omega)$=-π/2)

(Ideal) Differentiating TDE

Dynamic equation $y(t) = T\,\dfrac{dx}{dt}$

Transfer function $W(S) = T \cdot S$



a b

Figure 1.16– Time characteristics of D-TDE

a – unit step response (transient function) $h(t) = T \cdot \delta(t)$,

b – unit impulse response (weight function) $w(t) = \dfrac{dh(t)}{dt} = T \begin{cases} 0, & t \neq 0 \\ \pm\infty, & t = 0 \end{cases}$

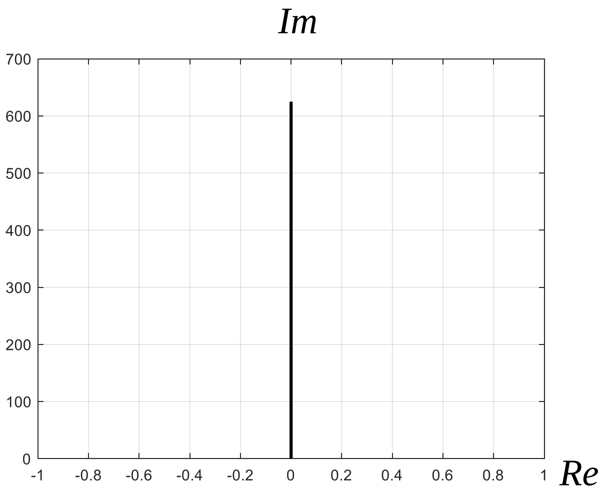


Figure 1.17 – Amplitude-phase characteristic of D-TDE

$$W(j\omega) = A(\omega)e^{j\varphi(\omega)} = Te^{j\frac{\pi}{2}} \text{ or } W(j\omega) = 0 + jT\omega$$

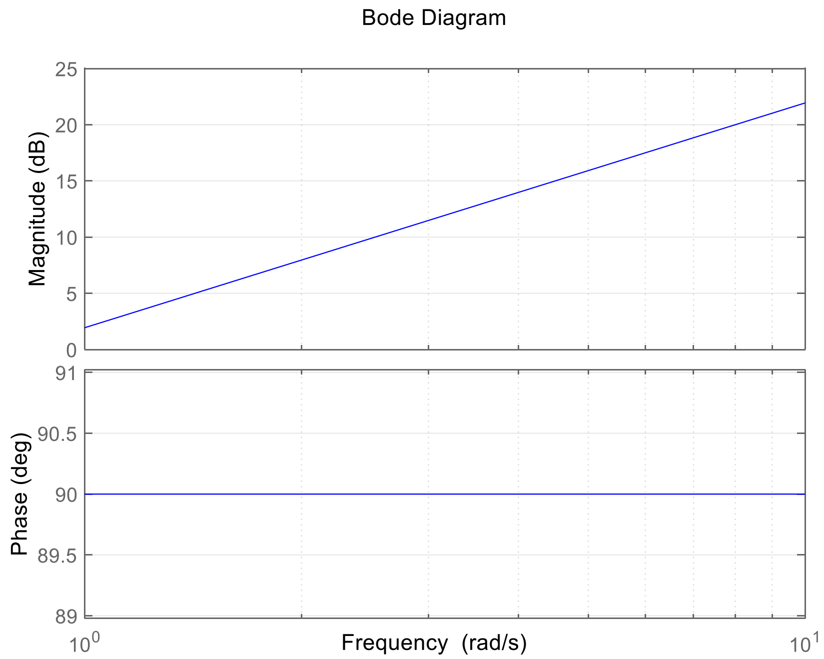


Figure 1.18 – Bode plot of D-TDE:
bode magnitude plot $L(\omega)=20\lg A(\omega)=20\lg T\omega$;
bode phase plot $\varphi(\omega)=\pi/2)$

<u>First-order aperiodic TDE</u>

Dynamic equation $T\dfrac{dy(t)}{dt}+y(t)=kx(t)$

Transfer function $W(S)=\dfrac{k}{TS+1}$

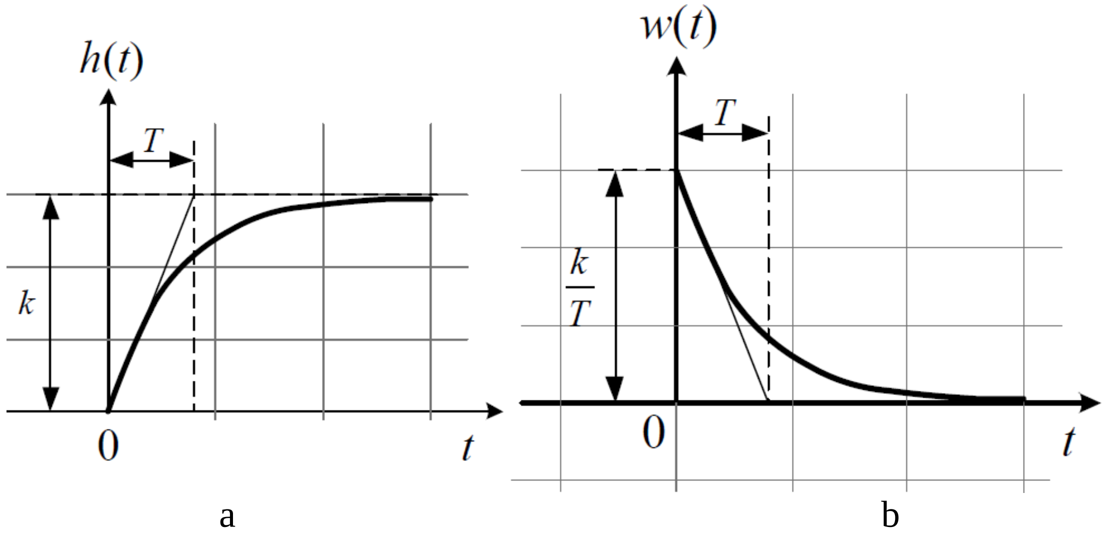


Figure 1.19 – Time characteristics of A-I-TDE

a – unit step response (transient function) $h(t)=k\left(1-e^{-\frac{t}{T}}\right)$,

b – unit impulse response (weight function) $w(t)=\dfrac{dh(t)}{dt}=\dfrac{k}{T}e^{-\frac{t}{T}}$

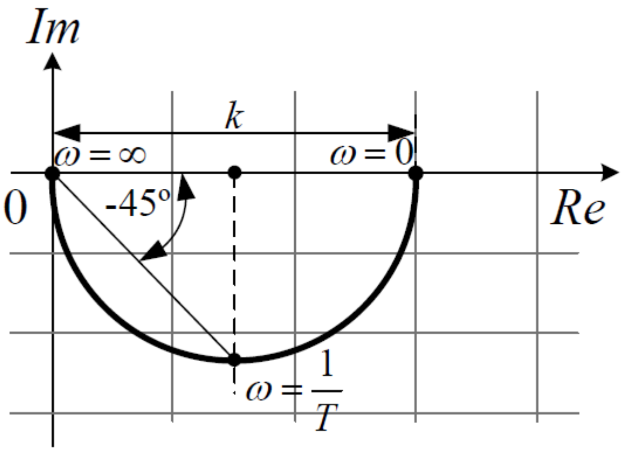


Figure 1.20 – Amplitude-phase characteristic of A-I-TDE

$$W(j\omega) = A(\omega)e^{j\varphi(\omega)} = \frac{k}{\sqrt{(T\omega)^2+1}}e^{jarctg\,(T\omega)} \text{ or } W(j\omega) = \frac{k}{(T\omega)^2+1} - j\frac{kT\omega}{(T\omega)^{+}+1}$$

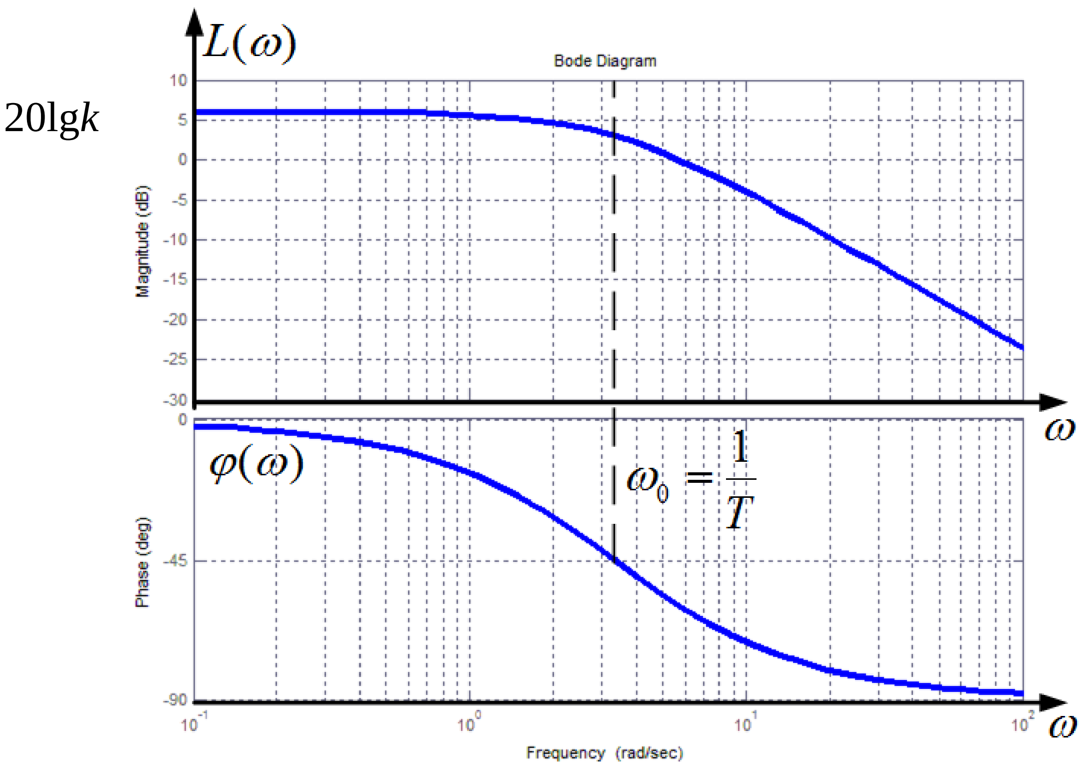


Figure 1.21 – Bode plot of A-I-TDE:

bode magnitude plot $L(\omega)$=20lg$A(\omega)$ $= 20\lg\left(\frac{k}{\sqrt{(T\omega)^2+1}}\right)$;

bode phase plot $\varphi(\omega)$=arctg(-Tω))

Link to the site with tutorial on Control Systems that may help
https://www.tutorialspoint.com/control_systems/index.htm